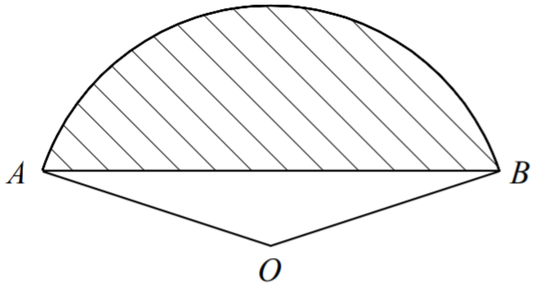
Practice Exam Paper D
Time: 2 Hours

P1 P2

1. The figure shows the sector OAB of a circle, centre O, in which $\angle AOB = 2.5$ radians.



Given that the perimeter of the sector is 36 cm,

a. Find the length OA, **(2)**

b. Find the area of the shaded segment **(3)**

(Total Marks: 5)

2. The second and fifth terms of a geometric series are −48 and 6 respectively.

a. Find the first term and the common ratio of the series. **(5)**

b. Find the sum to infinity of the series. **(2)**

c. Show that the difference between the sum of the first n terms of the series and its sum to infinity is given by 2^{6-n} . **(5)**

(Total Marks: 12)

3. A curve has the equation

$$4 \cos x + 2 \sin y = 3$$

a. Show that $\frac{dy}{dx} = 2 \sin x \sec y$. **(5)**

b. Find an equation for the tangent to the curve at the point $\left(\frac{\pi}{3}, \frac{\pi}{6}\right)$, giving your answer in the form $ax + by = c$, where a and b are integers. **(3)**

(Total Marks: 8)

4. Express $\frac{2+20x}{1+2x-8x^2}$ as a sum of partial fractions. **(4)**

b. Hence find the series expansion of $\frac{2+20x}{1+2x-8x^2}$, $|x| < \frac{1}{4}$, in ascending powers of x up to and including the term in x^3, simplifying each coefficient. **(5)**

(Total Marks: 9)

5. The functions f and g are defined by,

$$f(x) = 6x - 1$$
$$g(x) = \log_2(3x + 1)$$

a. Evaluate $gf(1)$. **(2)**

b. Find an expression for $g^{-1}(x)$. **(3)**

c. Find, in terms of natural logarithms, the solution of the equation $fg^{-1}(x) = 2$ **(4)**

(Total Marks: 9)

6a. Use the identities for cos (A + B) and cos (A − B) to prove that.

$$\cos P - \cos Q = -2 \sin \frac{P+Q}{2} \sin \frac{P-Q}{2}$$ **(4)**

b. Hence find all solutions in the interval $0 \leq x < 180$ to the equation $\cos 5x° + \sin 3x° - \cos x° = 0$. **(7)**

(Total Marks: 11)

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7. The function f is defined by,

$$f(x) \equiv x^2 - 2ax$$

where a is a positive constant

a. Showing the coordinates of any points where each graph meets the axes, sketch on separate diagrams the graphs of

i. $y = |f(x)|$
ii. $y = f(|x|)$ **(6)**

The function g is defined by $g(x) \equiv 3ax$,

b. Find $fg(a)$ in terms of a. **(2)**

c. Solve the equation

$$gf(x) = 9a^3$$ **(4)**

(Total Marks: 12)

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8. $f(x) = 2x + \sin x - 3 \cos x$

a. Show that the equation $f(x) = 0$ has a root in the interval [0.7, 0.8] **(2)**

b. Find an equation for the tangent to the curve $y = f(x)$ at the point where it crosses the y-axis. **(4)**

c. Find the values of the constants a, b and c, where $b > 0$ and $0 < c < \frac{\pi}{2}$, such that

$$f'(x) = a + b \cos (x - c).$$ **(4)**

d. Hence find the x-coordinates of the stationary points of the curve $y = f(x)$ in the interval $0 \leq x \leq 2\pi$, giving your answers to 2 decimal places. **(4)**

(Total Marks: 14)

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9. A curve has the equation

$$2x^2 + xy - y^2 + 18 = 0.$$

Find the coordinates of the points where the tangent to the curve is parallel to the x-axis. **(8)**

(Total Marks: 8)

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10. Use the substitution, $x = 2\tan u$, to show that,

$$\int_0^2 \frac{x^2}{x^2+4}\,dx = \frac{1}{2}(4 - \pi)$$ **(8)**

(Total Marks: 8)

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11. A curve has parametric equations

$$x = \frac{t}{2-t}, \qquad y = \frac{1}{1+t}, \qquad -1 < t < 2$$

a. Show that $\frac{dy}{dx} = -\frac{1}{2}\left(\frac{2-t}{1+t}\right)^2$ **(4)**

b. Find an equation for the normal to the curve at the point where $t = 1$. **(3)**

c. Show that the cartesian equation of the curve can be written in the form, $y = \frac{1+x}{1+3x}$ **(4)**

(Total Marks: 11)

12a. Use the trapezium rule with two intervals of equal width to find an estimate for the value of the integral,

$$\int_0^3 e^{\cos x}\,dx$$

giving your answer to 3 significant figures. **(5)**

b. Use the trapezium rule with four intervals of equal width to find another estimate for the value of the integral to 3 significant figures. **(2)**

c. Given that the true value of the integral lies between the estimates made in parts (a) and (b), comment on the shape of the curve $y = e^{\cos x}$ in the interval $0 \le x \le 3$ and explain your answer. **(2)**

(Total Marks: 9)

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13. Relative to a fixed origin, O, the points A and B have position vectors $\begin{pmatrix} 1 \\ 5 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ 3 \\ -6 \end{pmatrix}$ respectively.

Find, in exact, simplified form, the cosine of $\angle AOB$ **(4)**

(Total Marks: 4)

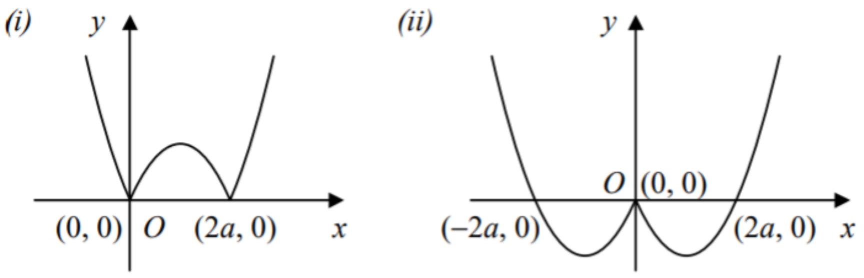
Total Marks: 120

Mark Scheme

1a	$P = 2r + (r \times 2.5) = \frac{9}{2}r = 36$	**M1**
	$OA = r = 8$ cm	**A1**
1b	$= \left(\frac{1}{2} \times 8^2 \times 2.5\right) - \left(\frac{1}{2} \times 8^2 \times \sin 2.5\right) = 60.8\ cm^2$	**M2** **A1**
2a	$ar = -48$ $ar^4 = 6$	**B1**
	$r^3 = \frac{6}{-48} = -\frac{1}{8}$	**M1**
	$r = \sqrt[3]{-\frac{1}{8}} = \frac{1}{2}$	**M1** **A1**
	$a = \frac{-48}{-\frac{1}{2}} = 96$	**A1**
2b	$\frac{96}{1-(-\frac{1}{2})} = 64$	**M1** **A1**
2c	$S_n = \frac{96\left[1-(-\frac{1}{2})^n\right]}{1-\left(-\frac{1}{2}\right)} = 64\left[1 - \left(-\frac{1}{2}\right)^n\right]$	**M1** **A1**
	$S_\infty - S_n = 64 - 64[1 - (-\frac{1}{2})^n]$	**M1**
	$= 64(-\frac{1}{2})^n = 2^6 \times (-1)^n \times 2^{-n} = (-1)^n \times 2^{6-n}$	**M1**
	Difference is magnitude, Therefore $= 2^{6-n}$	**A1**
3a	$-4 \sin x + (2 \cos y)\frac{dy}{dx} = 0$	**M1** **A2**
	$\frac{dy}{dx} = \frac{4 \sin x}{2 \cos y} = \frac{2 \sin x}{\cos y} = 2 \sin x \sec y$	**M1** **A1**
3b	Gradient $= 2 \times \frac{\sqrt{3}}{2} \times \frac{2}{\sqrt{3}} = 2$	**B1**
	$y - \frac{\pi}{6} = 2\left(x - \frac{\pi}{3}\right)$	**M1**
	$6y - \pi = 12x - 4\pi$ $4x - 2y = \pi$	**A1**
4a	$\frac{2+20x}{1+2x-8x^2} = \frac{2+20x}{(1-2x)(1+4x)} = \frac{A}{1-2x} + \frac{B}{1+4x}$	**B1**
	$2 + 20x = A(1 + 4x) + B(1 - 2x)$	**M1**
	$x = \frac{1}{2}$ $12 = 3A$ $A = 4$	**A1**
	$x = -\frac{1}{4}$ $-3 = \frac{3}{2}B$ $B = -2$	**A1**
4b	$\frac{2+20x}{1+2x-8x^2} = 4(1 - 2x)^{-2} - 2(1 + 4x)^{-1}$ $(1 - 2x)^{-1} = 1 + (-1)(-2x) + \frac{(-1)(-2)}{2}(-2x)^2 + \frac{(-1)(-2)(-3)}{3\times2}(-2x)^3 + \ldots$	**M1**
	$= 1 + 2x + 4x^2 + 8x^3 + \ldots$	**A1**
	$(1 + 4x)^{-1} = 1 + (-1)(4x) + \frac{(-1)(-2)}{2}(4x)^2 + \frac{(-1)(-2)(-3)}{3\times2}(4x)^3 + \ldots$ $= 1 - 4x + 16x^2 - 64x^3$	**A1**
	$\frac{2+20x}{1+2x-8x^2} = 4(1 + 2x + 4x^2 + 8x^3 + \ldots) - 2(1 - 4x + 16x^2 - 63x^3 + \ldots)$	**M1**
	$= 2 + 16x - 16x^2 + 160x^3 + \ldots$	**A1**

5a	g(5) = $\log_2 16 = 4$	**M1** **A1**
5b	$y = \log_2(3x + 1)$ $3x + 1 = 2^y$	**M1**
	$x = \frac{1}{3}(2^y - 1)$ $g(x) = \frac{1}{3}(2^x - 1)$	**M1** **A1**
5c	$fg^{-1}(x) = f[\frac{1}{3}(2^x - 1)] = 2(2^x - 1) - 1$	**M1**
	$= 2(2^x) - 3 = 2$ $2^x = \frac{5}{2}$	**A1**
	$x = \frac{\ln\left(\frac{5}{2}\right)}{ln2}$	**M1** **A1**

6a	$\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$ $\cos(A - B) \equiv \cos A \cos B + \sin A \sin B$ subtracting, $\cos(A + B) - \cos(A - B) \equiv -2 \sin A \sin B$	**M1** **A1**
	let $P = A + B$, $Q = A - B$ adding, $P + Q = 2A \Rightarrow A = \frac{P+Q}{2}$	**M1**
	subtracting, $P - Q = 2B \Rightarrow B = \frac{P-Q}{2}$ Therefore, $\cos P - \cos Q \equiv -2 \sin \frac{P+Q}{2} \sin \frac{P-Q}{2}$	**A1**
6b	$(\cos 5x - \cos x) + \sin 3x = 0$ $-2 \sin 3x \sin 2x + \sin 3x = 0$	**M1**
	$\sin 3x(1 - 2 \sin 2x) = 0$	**M1**
	$\sin 3x = 0$ $\sin 2x = ½$	**A1**
	$3x = 0, 180, 360$ $2x = 30, 150$	**B1**
	$x = 0, 15, 60, 75, 120$	**M1** **A2**

7a		**B3** **B3**
7b	$= f(3a^2) = 9a^4 - 6a^3$	**M1** **A1**
7c	$gf(x) = 3a(x^2 - 2ax)$	**M1**
	$3a(x^2 - 2ax) = 9a^3$ $x^2 - 2ax - 3a^2 = 0$	**A1**
	$(x + a)(x - 3a) = 0$	**M1**
	$x = -a$ $x = 3a$	**A1**

8a	$f(0.7) = -0.25$, $f(0.8) = 0.23$	**M1**
	sign change, $f(x)$ continuous ∴ root	**A1**
8b	$f'(x) = 2 + \cos x + 3 \sin x$	**M1**
	$x = 0$, $y = -3$, $grad = 3$	**A1**
	∴ $y = 3x - 3$	**M1**

		A1
8c	$\cos x + 3\sin x = b\cos x\cos c + b\sin x\sin c$ $b\cos c = 1$, $b\sin c = 3$ $b = \sqrt{1^2+3^2} = \sqrt{10}$	**M1**
	$\tan c = 3$, $c = 1.25$ (3sf)	**M1**
	$\therefore a = 2$, $b = 10$, $c = 1.25$	**A2**
8d	SP: $2 + \sqrt{10}\cos(x - 1.249) = 0$ $\cos(x - 1.249) = -\frac{2}{\sqrt{10}}$	**M1**
	$x - 1.249 = \pi - 0.8861, \pi + 0.8861 = 2.256, 4.028$	**M1**
	$x = 3.50, 5.28$ (2dp)	**A2**

9	$4x + y + x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$	**M1** **A2**
	S.P: $\frac{dy}{dx} = 0$ $4x + y = 0$ $y = -4x$	**M1** **A1**
	Subbing in, $2x^2 - 4x^2 - 16x^2 + 18 = 0$	**M1**
	$x^2 = 1$ $x = \pm 1$ Therefore (-1, 4), (1, -4)	**A2**

10	$x = \tan u$ $\frac{dx}{du} = 2\sec^2 u$	**M1**
	$x = 0, u = 0$ $x = 2, u = \frac{\pi}{4}$	**B1**
	$I = \int_0^{\frac{\pi}{4}} \frac{4\tan^2 u}{4\sec^2 u} \times 2\sec^2 u\,du = \int_0^{\frac{\pi}{4}} 2\tan^2 u\,du$	**A1**
	$\int_0^{\frac{\pi}{4}} (2\sec^2 u - 2)\,du$	**M1**
	$= [\tan u - 2u]_0^{\frac{\pi}{4}}$	**M1** **A1**
	$= (2 - \frac{\pi}{2}) - (0) = \frac{1}{2}(4 - \pi)$	**M1** **A1**

11a	$\frac{dx}{dt} = \frac{1\times(2-t) - t\times(-1)}{(2-t)^2} = \frac{2}{(2-t)^2}$ $\frac{dy}{dt} = -(1+t)^{-2}$	**M1** **B1**
	$\frac{dy}{dx} = -\frac{1}{(1+t)^2} \div \frac{2}{(2-t)^2} = -\frac{(2-t)^2}{2(1+t)^2} = -\frac{1}{2}\left(\frac{2-t}{1+t}\right)^2$	**M1** **A1**
11b	$t = 1$, $x = 1$, $y = \frac{1}{2}$, grad = $-\frac{1}{8}$ grad of normal = 8	**B1**
	$y - \frac{1}{2} = 8(x - 1)$	**M1** **A1**
11c	$x(2 - t) = t$	**M1**
	$2x = t(1 + x)$ $t = \frac{2x}{1+x}$	**A1**
	$y = \frac{1}{1+\frac{2x}{1+x}} = \frac{1+x}{(1+x)+2x}$ $y = \frac{1+x}{1+3x}$	**M1** **A1**

<table>
<tr><td rowspan="3">12a</td><td><table><tr><td>x</td><td>0</td><td>0.75</td><td>1.5</td><td>2.25</td><td>3</td></tr><tr><td>y</td><td>2.7183</td><td>2.0786</td><td>1.0733</td><td>0.5336</td><td>0.3716</td></tr></table></td><td>B2</td></tr>
<tr><td>$= \frac{1}{2} \times 1.5 \times [2.7183 + 0.3716 + 2(1.0733)]$</td><td>B1
M1</td></tr>
<tr><td>= 3.93 (3sf)</td><td>A1</td></tr>
<tr><td rowspan="2">12b</td><td>$= \frac{1}{2} \times 0.75 \times [2.7183 + 0.3716 + 2(2.0786 + 1.0733 + 0.5336)]$</td><td>M1</td></tr>
<tr><td>= 3.92 (3sf)</td><td>A1</td></tr>
<tr><td>12c</td><td>Curve must be above top of trapezia in some places and below in others hence position of ordinates determines whether estimate is high or low</td><td>B2</td></tr>
</table>

<table>
<tr><td rowspan="2">13</td><td>$\left|\frac{1\times6+5\times3+(-1)\times(-6)}{\sqrt{1+25+1}\times\sqrt{36+9+36}}\right|$</td><td>M1
A1</td></tr>
<tr><td>$= \frac{27}{\sqrt{27}\times\sqrt{81}} = \frac{\sqrt{27}}{9} = \frac{3\sqrt{3}}{9}$
$= \frac{1}{3}\sqrt{3}$</td><td>M1
A1</td></tr>
</table>

Topic List

Q1	Radians
Q2	Sequences and series
Q3	Differentiation
Q4	Partial fractions, binomial series
Q5	Functions
Q6	Trigonometry
Q7	Functions
Q8	Numerical methods, differentiation, trig.
Q9	Differentiation
Q10	Integration
Q11	Parametric equations
Q12	Trapezium rule
Q13	Vectors