Practice Exam Paper C
Time: 2 Hours

P1 P2

1. A curve has the equation,

$$x^2(2 + y) - y^2 = 0$$

Find an expression for $\frac{dy}{dx}$ in terms of x and y **(6)**

(Total Marks: 6)

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2. $f(x) = \frac{3}{\sqrt{1-x}}, |x| < 1$

a. Show that $f(\frac{1}{10}) = \sqrt{10}$ **(2)**

b. Expand $f(x)$ in ascending powers of x up to and including the term in x^3 , simplifying each coefficient. **(3)**

c. Use your expansion to find an approximate value for $\sqrt{10}$, giving your answer to 8 significant figures. **(1)**

d. Find, to 1 significant figure, the percentage error in your answer to part (*c*) **(2)**

(Total Marks: 8)

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3. Express $\sqrt{3}$ sin θ + cos θ in the form R sin $(\theta + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$ **(4)**

b. State the maximum value of $\sqrt{3}$ sin θ + cos θ and the smallest positive value of θ for which this maximum value occurs. **(1)**

c. Solve the equation,

$$\sqrt{3} \sin \theta + \cos \theta + \sqrt{3} = 0$$

for θ in the interval, $-\pi \leq \theta \leq \pi$, giving your answers in terms of π **(5)**

(Total Marks: 10)

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4. A curve has the equation $y = (3x - 5)^3$

a. Find an equation for the tangent to the curve at the point $P(2, 1)$ **(4)**

The tangent to the curve at the point Q is parallel to the tangent at P.

b. Find the coordinates of Q **(3)**

(Total Marks: 7)

5a. Use the identities for cos ($A + B$) and cos ($A - B$) to prove that,

$$2 \cos A \cos B \equiv \cos (A + B) + \cos (A - B)$$

(2)

b. Hence or otherwise, find in terms of π the solutions of the equation,

$$2\cos\left(x + \frac{\pi}{2}\right) = \sec\left(x + \frac{\pi}{6}\right)$$

for x in the interval $0 \le x \le \pi$ **(7)**

(Total Marks: 9)

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6. Show that $(2x + 3)$ is a factor of $(2x^3 - x^2 + 4x + 15)$ **(2)**

b. Hence, simplify, $\frac{2x^2+x-3}{2x^3-x^2+4x+15}$ **(4)**

c. Find the coordinates of the stationary points of the curve with equation, $y = \frac{2x^2+x-3}{2x^3-x^2+4x+15}$ **(6)**

(Total Marks: 12)

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7. The finite region R is bounded by the curve $y = 1 + 3\sqrt{x}$, the x-axis and the lines $x = 2$ and $x = 8$.

a. Use the trapezium rule with three intervals of equal width to estimate to 3 significant figures the area of R. **(6)**

b. Use integration to find the exact area of R in the form $a + b\sqrt{2}$. **(5)**

c. Find the percentage error in the estimate made in part (a) **(2)**

(Total Marks: 13)

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8. The first three terms of a geometric series are $(x - 2)$, $(x + 6)$ and x^2 respectively.

a. Show that x must be a solution of the equation

$$x^3 - 3x^2 - 12x - 36 = 0.$$ **(3)**

b. Verify that $x = 6$ is a solution of equation (a) and show that there are no other real solutions. **(6)**

Using $x = 6$,

c. Find the common ratio of the series, **(1)**

d. Find the sum of the first eight terms of the series **(2)**

(Total Marks: 12)

9a. Use the derivative of cos x to prove that, $\frac{d}{dx}(\sec x) = \sec x \tan x$ **(4)**

The curve C has the equation $y = e^{2x}\sec x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

b. Find an equation for the tangent to C at the point where it crosses the y-axis. **(4)**

c. Find, to 2 decimal places, the x-coordinate of the stationary point of C. **(3)**

(Total Marks: 11)

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10. A curve has the equation $y = \frac{e^2}{x} + e^x$, $x \neq 0$

a. Find $\frac{dy}{dx}$ **(2)**

b. Show that the curve has a stationary point in the interval [1.3, 1.4]. **(3)**

The point *A* on the curve has *x*-coordinate 2.

c. Show that the tangent to the curve at *A* passes through the origin. **(4)**

The tangent to the curve at *A* intersects the curve again at the point *B*.
The *x*-coordinate of *B* is to be estimated using the iterative formula

$$x_{n+1} = -\frac{2}{3}\sqrt{3 + 3x_n e^{x_n - 2}}$$

when $x_0 = -1$

d. Find x_1, x_2 and x_3 to 7 significant figures and hence state the *x*-coordinate of *B* to 5 significant figures. **(4)**

(Total Marks: 13)

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11. $f(x) = x^2 - 2x + 5$, $x \geq 1$

a. Express $f(x)$ in the form $(x + a)^2 + b$, where *a* and *b* are constants **(2)**

b. State the range of *f*. **(1)**

c. Find an expression for $f^{-1}(x)$. **(3)**

d. Describe fully two transformations that would map the graph of $y = f^{-1}(x)$ onto the graph of $y = \sqrt{x}$, $x \geq 0$. **(2)**

e. Find an equation for the normal to the curve $y = f^{-1}(x)$ at the point where $x = 8$ **(4)**

(Total Marks: 12)

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12. Find, in terms of π, the values of y in the interval $0 \leq y < 2\pi$ for which,

$$2 \sin y = \tan y$$ **(7)**

(Total Marks: 7)

Total Marks: 120

Mark Scheme

1	$2x(x+y) + x^2\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$	**M2** **A2**
	$\frac{dy}{dx} = \frac{2x(2+y)}{2y-x^2}$	**M1** **A1**

2a	$f(\frac{1}{10}) = \frac{3}{\sqrt{1-\frac{1}{10}}} = \frac{3}{\sqrt{\frac{9}{10}}} = \sqrt{10}$	**M1** **A1**
2b	$= 3(1-x)^{-\frac{1}{2}} = 3\left[1 + \left(-\frac{1}{2}\right)(-x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}(-x)^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3\times 2}(-x)^3 + \cdots\right.$	**M1**
	$= 3 + \frac{3}{2}x + \frac{9}{8}x^2 + \frac{15}{16}x^3$	**A2**
2c	$\sqrt{10} = f\left(\frac{1}{10}\right) \sim 3 + \frac{3}{20} + \frac{9}{800} + \frac{15}{16000} = 3.1621875$	**B1**
2d	$= \frac{\sqrt{10} - 3.1621875}{\sqrt{10}} \times 100\% = 0.003\%$	**M1** **A1**

3a	$\sqrt{3}\sin\theta + \cos\theta = R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$ $R\cos\alpha = \sqrt{3}$ $R\sin\alpha = 1$ $R = \sqrt{3+1} = 2$	**M1** **A1**
	$\tan\alpha = \frac{1}{\sqrt{3}}$ $\alpha = \frac{\pi}{6}$	**M1** **A1**
3b	Maximum = 2	**B1**
	Occurs when, $\theta + \frac{\pi}{6} = \frac{\pi}{2}$ $\theta = \frac{\pi}{3}$	**M1** **A1**
3c	$2\sin(\theta + \frac{\pi}{6}) + \sqrt{3} = 0$ $\sin(\theta + \frac{\pi}{6}) = -\frac{\sqrt{3}}{2}$	**M1**
	$\theta + \frac{\pi}{6} = -\frac{\pi}{3}$ $-\pi + \frac{\pi}{3} = -\frac{\pi}{3}, -\frac{2\pi}{3}$	**B1** **M1**
	$\theta = -\frac{5\pi}{6}, -\frac{\pi}{2}$	**A2**

4a	$\frac{dy}{dx} = 3(3x-5)^5 \times 3 = 9(3x-5)^2$	**M1**
	gradient = 9	**A1**
	$y - 1 = 9(x - 2)$	**M1** **A1**
4b	$9(3x-5)^2 = 9$ $3x - 5 = \pm 1$	**M1**
	$x = 2$ (at P) $x = \frac{4}{3}$	**A1**
	Therefore Q: $(\frac{4}{3}, -1)$	**A1**

5a	$\cos(A+B) \equiv \cos A\cos B - \sin A\sin B$ $\cos(A-B) \equiv \cos A\cos B + \sin A\sin B$ adding, $2\cos A\cos B = \cos(A+B) + \cos(A-B)$	**M1** **A1**
5b	$2\cos(x + \frac{\pi}{2})\cos\left(x + \frac{\pi}{6}\right) = 1$	**M1**

	$\cos(2x + \frac{2\pi}{3}) + \cos\frac{\pi}{3} = 1$	**M1**
	$\cos(2x + \frac{2\pi}{3}) = 1 - \frac{1}{2} = \frac{1}{2}$	**A1**
	$2x + \frac{2}{3}\pi = 2\pi - \frac{\pi}{3}$ $2\pi + \frac{\pi}{3} = \frac{5\pi}{3}, \frac{7\pi}{3}$	**B1**
	$2x = \pi, \frac{5\pi}{3}$	**M1**
	$x = \frac{\pi}{2}, \frac{5\pi}{6}$	**A2**

6a	let $f(x) = 2x^3 - x^2 + 4x + 15$ $f(-\frac{3}{2}) = -\frac{27}{4} - \frac{9}{4} - 6 + 16 = 0$ Therefore $(2x + 3)$ is a factor.	**M1** **A1**
6b	$x^2 - 2x + 5$ $2x + 3 \overline{)\,2x^3 - x^2 + 4x + 15}$ $2x^3 + 3x^2$ $- 4x^2 + 4x$ $- 4x^2 - 6x$ $10x + 15$ $10x + 15$	**M1** **A1**
	Therefore, $f(x) = (2x + 3)(x^2 - 2x + 5)$ $\frac{2x^2+x-3}{2x^3-x^2+4x+15} = \frac{(2x+3)(x-1)}{(2x+3)(x^2-2x+5)}$	**M1** **A1**
6c	$\frac{dy}{dx} = \frac{1\times(x^2-2x+5)-(x-1)(2x-2)}{(x^2-2x+5)^2} = \frac{-x^2+2x+3}{(x^2-2x+5)^2}$	**M1** **A2**
	Stationary point, $\frac{-x^2+2x+3}{(x^2-2x+5)^2} = 0$ $-x^2 + 2x + 3 = 0$ $-(x + 1)(x - 3) = 0$	**M1**
	$x = -1$ $x = 3$ Therefpre, $(-1, -\frac{1}{4}), (3, \frac{1}{4})$	**M1**

7a	(table below)	**M1** **A1**

x	2	4	6	8
$1 + 3\sqrt{x}$	5.243	8.348	8.348	9.485

	Area $\approx \frac{1}{2} \times 2 \times [5.243 + 9.485 + 2(7 + 8.348)]$	**B1** **M1** **A1**
	= 45.4 (3 s.f)	**A1**
7b	$\int_2^8 (1 + 3\sqrt{x})dx$ $= [x + 2x^{\frac{3}{2}}]_2^8$	**M1** **A1**
	$[8 + 2(2\sqrt{2})^3] - [2 + 2(2\sqrt{2})]$	**M1**
	$= (8 + 32\sqrt{2}) - (2 + 4\sqrt{2})$	**M1**
	$= 6 + 28\sqrt{2}$	**A1**
7c	$\frac{6+28\sqrt{2}-45.4}{6+28\sqrt{2}} \times 100\% = 0.43\%$	**M1** **A1**

8a	$r = \frac{x+6}{x-2} = \frac{x^2}{x+6}$	**M1**
	$(x + 6)^2 = x^2(x - 2)$	**M1**
	$x^2 + 12x + 36 = x^3 - 2x^2$	**A1**

	$x^3 - 3x^2 - 12x - 36 = 0$	
8b	When $x = 6$, LHS = 216 – 108 – 72 – 36 = 0 Therefore $x = 6$	**B1**
	$x^2 + 3x + 6$ $x - 6 \overline{)\, x^3 - 3x^2 - 12x - 36}$ $x^3 - 6x^2$ $3x^2 - 12x$ $3x^2 - 18x$ $6x - 36$ $6x - 36$	**M1** **A1**
	$(x - 6)(x^2 + 3x + 6) = 0$ $x = 6$ $b^2 - 4ac = 3^2 - (4 \times 1 \times 6) = -15$	**M1** **A1**
	$b^2 - 4ac < 0$ Therefore, no real solutions to quadratic Therefore, no other solutions	**A1**
8c	$r = \frac{6+6}{6-2} = 3$	**B1**
8d	$a = 6 - 2 = 4$ $S_8 = \frac{4(3^8-1)}{301} = 13120$	**M1** **A1**

9a	$\frac{d}{dx}(\sec x) = \frac{d}{dx}[(\cos x)^{-1}]$ $= -(\cos x)^{-2} \times (-\sin x)$	**M1** **A1**
	$= \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x} \times \frac{\sin x}{\cos x}$	**M1**
	$= \sec x \tan x$	**A1**
9b	$\frac{dy}{dx} = 2e^{2x} \times \sec x + e^{2x} \times \sec x \tan x \ = e^{2x} \sec x\,(2 + \tan x)$	**M1** **A1**
	$x = 0$ $y = 1$ gradient = 2	**M1**
	$y = 2x + 1$	**A1**
9c	SP: $e^{2x} \sec x\,(2 + \tan x) = 0$	**M1**
	$\tan x = -2$	**M1**
	$x = -1.11$ (2 d.p)	**A1**

10a	$\frac{dy}{dx} = -e^2x^{-2} + e^x$	**M1** **A1**
10b	SP: $-e^2x^{-2} + e^x = 0$	**M1**
	Let $f(x) = -e^2x^{-2} + e^x$ $f(1.3) = -0.70$ $f(1.4) = 0.29$	**M1**
	Sign change, $f(x)$ continuous therefore root.	**A1**
10c	$x = 2$ $y = \frac{3}{2}e^2$ Gradient = $\frac{3}{4}e^2$	**M1**
	$y - \frac{3}{2}e^2 = \frac{3}{4}e^2(x - 2)$	**M1** **A1**
	$y = \frac{3}{4}e^2x$ $x = 0$ $y = 0$ So passes through origin	**A1**
10d	$x_1 = -1.125589$ $x_2 = -1.125803$	**M1** **A2**

	$x_3 = -1.125804$	
	x – coordinate of B = -1.1258 (5 s.f)	**A1**

11a	$f(x) = (x-1)^2 - 1 + 5 = (x-1)^2 + 4$	**M1** **A1**
11b	$f(x) \geq 4$	**B1**
11c	$y = (x-1)^2 + 4$ $(x-1)^2 = y - 4$ $x - 1 = \pm\sqrt{y-4}$ $x = \pm\sqrt{y-4} + 1$	**M1**
	$f^{-1}(x) = 1 + \sqrt{x+4}$	**M1** **A1**
11d	Translation by 4 units in negative x direction Translation by 1 unit in negative y direction (either first)	**B2**
11e	$\frac{dy}{dx} = \frac{1}{2}(x-4)^{-\frac{1}{2}}$	**M1**
	$x = 8$ $y = 3$ gradient = $\frac{1}{4}$	**A1**
	Gradient of normal = - 4 $y - 3 = -4(x - 8)$ $y = 35 - 4x$	**M1** **A1**

12	$2\sin y\cos y = \sin y$	**M1**
	$\sin y\,(2\cos y - 1) = 0$	**M1**
	$\sin y = 0$ $\cos y = \frac{1}{2}$	**A1**
	$y = 0, \pi$ or $\frac{\pi}{3}, 2\pi - \frac{\pi}{3}$	**B1** **M1**
	$y = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$	**A2**

Topic List

Q1	Implicit differentiation
Q2	Binomial expansion
Q3	Trig equations
Q4	Chain rule differentiation
Q5	Addition formulae
Q6	Factor theorem
Q7	Trapezium rule
Q8	Geometric series
Q9	Differentiation, stationary points
Q10	Stationary points, iteration
Q11	Differentiation, normal, completing the square
Q12	Solving trig equations