



# Practice Exam Paper C

Time: 2 Hours

P1

P2

1. A curve has the equation,

$$x^2(2 + y) - y^2 = 0$$

Find an expression for  $\frac{dy}{dx}$  in terms of  $x$  and  $y$

(6)

**(Total Marks: 6)**

2.  $f(x) = \frac{3}{\sqrt{1-x}}$ ,  $|x| < 1$

a. Show that  $f\left(\frac{1}{10}\right) = \sqrt{10}$

(2)

b. Expand  $f(x)$  in ascending powers of  $x$  up to and including the term in  $x^3$ , simplifying each coefficient. (3)

c. Use your expansion to find an approximate value for  $\sqrt{10}$ , giving your answer to 8 significant figures. (1)

d. Find, to 1 significant figure, the percentage error in your answer to part (c) (2)

**(Total Marks: 8)**

3. Express  $\sqrt{3} \sin \theta + \cos \theta$  in the form  $R \sin(\theta + \alpha)$  where  $R > 0$  and  $0 < \alpha < \frac{\pi}{2}$  (4)

b. State the maximum value of  $\sqrt{3} \sin \theta + \cos \theta$  and the smallest positive value of  $\theta$  for which this maximum value occurs. (1)

c. Solve the equation,

$$\sqrt{3} \sin \theta + \cos \theta + \sqrt{3} = 0$$

for  $\theta$  in the interval,  $-\pi \leq \theta \leq \pi$ , giving your answers in terms of  $\pi$  (5)

**(Total Marks: 10)**

4. A curve has the equation  $y = (3x - 5)^3$

a. Find an equation for the tangent to the curve at the point  $P(2, 1)$  (4)

The tangent to the curve at the point  $Q$  is parallel to the tangent at  $P$ .

b. Find the coordinates of  $Q$  (3)

**(Total Marks: 7)**

5a. Use the identities for  $\cos(A + B)$  and  $\cos(A - B)$  to prove that,

$$2 \cos A \cos B \equiv \cos(A + B) + \cos(A - B)$$

(2)

b. Hence or otherwise, find in terms of  $\pi$  the solutions of the equation,

$$2 \cos \left(x + \frac{\pi}{2}\right) = \sec \left(x + \frac{\pi}{6}\right)$$

for  $x$  in the interval  $0 \leq x \leq \pi$

(7)

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**(Total Marks: 9)**

6. Show that  $(2x + 3)$  is a factor of  $(2x^3 - x^2 + 4x + 15)$

(2)

b. Hence, simplify,  $\frac{2x^2+x-3}{2x^3-x^2+4x+15}$

(4)

c. Find the coordinates of the stationary points of the curve with equation,  $y = \frac{2x^2+x-3}{2x^3-x^2+4x+15}$

(6)

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**(Total Marks: 12)**

7. The finite region  $R$  is bounded by the curve  $y = 1 + 3\sqrt{x}$ , the  $x$ -axis and the lines  $x = 2$  and  $x = 8$ .

a. Use the trapezium rule with three intervals of equal width to estimate to 3 significant figures the area of  $R$ .

(6)

b. Use integration to find the exact area of  $R$  in the form  $a + b\sqrt{2}$ .

(5)

c. Find the percentage error in the estimate made in part (a)

(2)

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**(Total Marks: 13)**

8. The first three terms of a geometric series are  $(x - 2)$ ,  $(x + 6)$  and  $x^2$  respectively.

a. Show that  $x$  must be a solution of the equation

$$x^3 - 3x^2 - 12x - 36 = 0.$$

(3)

b. Verify that  $x = 6$  is a solution of equation (a) and show that there are no other real solutions.

(6)

Using  $x = 6$ ,

c. Find the common ratio of the series,

(1)

d. Find the sum of the first eight terms of the series

(2)

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**(Total Marks: 12)**

9a. Use the derivative of  $\cos x$  to prove that,  $\frac{d}{dx}(\sec x) = \sec x \tan x$

(4)

The curve  $C$  has the equation  $y = e^{2x} \sec x$ ,  $-\frac{\pi}{2} < x < \frac{\pi}{2}$ .

b. Find an equation for the tangent to  $C$  at the point where it crosses the  $y$ -axis.

(4)

c. Find, to 2 decimal places, the  $x$ -coordinate of the stationary point of  $C$ .

(3)

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**(Total Marks: 11)**



10. A curve has the equation  $y = \frac{e^2}{x} + e^x$ ,  $x \neq 0$

a. Find  $\frac{dy}{dx}$  (2)

b. Show that the curve has a stationary point in the interval  $[1.3, 1.4]$ . (3)

The point  $A$  on the curve has  $x$ -coordinate 2.

c. Show that the tangent to the curve at  $A$  passes through the origin. (4)

The tangent to the curve at  $A$  intersects the curve again at the point  $B$ .

The  $x$ -coordinate of  $B$  is to be estimated using the iterative formula

$$x_{n+1} = \frac{2}{3} \sqrt{3 + 3x_n e^{x_n - 2}}$$

when  $x_0 = -1$

d. Find  $x_1$ ,  $x_2$  and  $x_3$  to 7 significant figures and hence state the  $x$ -coordinate of  $B$  to 5 significant figures. (4)

**(Total Marks: 13)**

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11.  $f(x) = x^2 - 2x + 5$ ,  $x \geq 1$

a. Express  $f(x)$  in the form  $(x + a)^2 + b$ , where  $a$  and  $b$  are constants (2)

b. State the range of  $f$ . (1)

c. Find an expression for  $f^{-1}(x)$ . (3)

d. Describe fully two transformations that would map the graph of  $y = f^{-1}(x)$  onto the graph of  $y = \sqrt{x}$ ,  $x \geq 0$ . (2)

e. Find an equation for the normal to the curve  $y = f^{-1}(x)$  at the point where  $x = 8$  (4)

**(Total Marks: 12)**

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12. Find, in terms of  $\pi$ , the values of  $y$  in the interval  $0 \leq y < 2\pi$  for which,

$$2 \sin y = \tan y \quad (7)$$

**(Total Marks: 7)**

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**Total Marks: 120**

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## Mark Scheme

<b>1</b>	$2x(x + y) + x^2 \frac{dy}{dx} - 2y \frac{dy}{dx} = 0$	<b>M2</b> <b>A2</b>
	$\frac{dy}{dx} = \frac{2x(2+y)}{2y-x^2}$	<b>M1</b> <b>A1</b>
<b>2a</b>	$f\left(\frac{1}{10}\right) = \frac{3}{\sqrt{1-\frac{1}{10}}} = \frac{3}{\sqrt{\frac{9}{10}}} = \sqrt{10}$	<b>M1</b> <b>A1</b>
<b>2b</b>	$= 3(1-x)^{-\frac{1}{2}} = 3 \left[ 1 + \left(-\frac{1}{2}\right)(-x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}(-x)^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3 \times 2}(-x)^3 + \dots \right]$	<b>M1</b>
	$= 3 + \frac{3}{2}x + \frac{9}{8}x^2 + \frac{15}{16}x^3$	<b>A2</b>
<b>2c</b>	$\sqrt{10} = f\left(\frac{1}{10}\right) \sim 3 + \frac{3}{20} + \frac{9}{800} + \frac{15}{16000} = 3.1621875$	<b>B1</b>
<b>2d</b>	$= \frac{\sqrt{10} - 3.1621875}{\sqrt{10}} \times 100\% = 0.003\%$	<b>M1</b> <b>A1</b>
<b>3a</b>	$\sqrt{3} \sin \theta + \cos \theta = R \sin \theta \cos \alpha + R \cos \theta \sin \alpha$ $R \cos \alpha = \sqrt{3}$ $R \sin \alpha = 1$ $R = \sqrt{3+1} = 2$	<b>M1</b> <b>A1</b>
	$\tan \alpha = \frac{1}{\sqrt{3}}$ $\alpha = \frac{\pi}{6}$	<b>M1</b> <b>A1</b>
<b>3b</b>	Maximum = 2	<b>B1</b>
	Occurs when, $\theta + \frac{\pi}{6} = \frac{\pi}{2}$ $\theta = \frac{\pi}{3}$	<b>M1</b> <b>A1</b>
<b>3c</b>	$2 \sin\left(\theta + \frac{\pi}{6}\right) + \sqrt{3} = 0$	<b>M1</b>
	$\sin\left(\theta + \frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$	
	$\theta + \frac{\pi}{6} = -\frac{\pi}{3}$	<b>B1</b>
	$-\pi + \frac{\pi}{3} = -\frac{\pi}{3}, -\frac{2\pi}{3}$	<b>M1</b>
	$\theta = -\frac{5\pi}{6}, -\frac{\pi}{2}$	<b>A2</b>
<b>4a</b>	$\frac{dy}{dx} = 3(3x-5)^5 \times 3 = 9(3x-5)^2$	<b>M1</b>
	gradient = 9	<b>A1</b>
	$y-1 = 9(x-2)$	<b>M1</b> <b>A1</b>
<b>4b</b>	$9(3x-5)^2 = 9$	<b>M1</b>
	$3x-5 = \pm 1$	
	$x = 2$ (at P)	<b>A1</b>
	$x = \frac{4}{3}$	<b>A1</b>
	Therefore Q: $\left(\frac{4}{3}, -1\right)$	<b>A1</b>
<b>5a</b>	$\cos(A+B) \equiv \cos A \cos B - \sin A \sin B$ $\cos(A-B) \equiv \cos A \cos B + \sin A \sin B$ adding, $2 \cos A \cos B = \cos(A+B) + \cos(A-B)$	<b>M1</b> <b>A1</b>
<b>5b</b>	$2 \cos\left(x + \frac{\pi}{2}\right) \cos\left(x + \frac{\pi}{6}\right) = 1$	<b>M1</b>

	$\cos(2x + \frac{2\pi}{3}) + \cos \frac{\pi}{3} = 1$	<b>M1</b>
	$\cos(2x + \frac{2\pi}{3}) = 1 - \frac{1}{2} = \frac{1}{2}$	<b>A1</b>
	$2x + \frac{2}{3}\pi = 2\pi - \frac{\pi}{3}$ $2\pi + \frac{\pi}{3} = \frac{5\pi}{3}, \frac{7\pi}{3}$	<b>B1</b>
	$2x = \pi, \frac{5\pi}{3}$	<b>M1</b>
	$x = \frac{\pi}{2}, \frac{5\pi}{6}$	<b>A2</b>

<b>6a</b>	let $f(x) = 2x^3 - x^2 + 4x + 15$ $f(\frac{3}{2}) = -\frac{27}{4} - \frac{9}{4} - 6 + 16 = 0$ Therefore $(2x + 3)$ is a factor.	<b>M1</b> <b>A1</b>
<b>6b</b>	$\begin{array}{r} x^2 - 2x + 5 \\ 2x+3 \overline{) 2x^3 - x^2 + 4x + 15} \\ \underline{2x^3 + 3x^2} \phantom{+ 4x + 15} \\ -4x^2 + 4x \phantom{+ 15} \\ \underline{-4x^2 - 6x} \phantom{+ 15} \\ 10x + 15 \\ \underline{10x + 15} \\ 0 \end{array}$ Therefore, $f(x) = (2x + 3)(x^2 - 2x + 5)$ $\frac{2x^2+x-3}{2x^3-x^2+4x+15} = \frac{(2x+3)(x-1)}{(2x+3)(x^2-2x+5)}$	<b>M1</b> <b>A1</b>
<b>6c</b>	$\frac{dy}{dx} = \frac{1 \times (x^2 - 2x + 5) - (x-1)(2x-2)}{(x^2 - 2x + 5)^2} = \frac{-x^2 + 2x + 3}{(x^2 - 2x + 5)^2}$ Stationary point, $\frac{-x^2 + 2x + 3}{(x^2 - 2x + 5)^2} = 0$ $-x^2 + 2x + 3 = 0$ $-(x+1)(x-3) = 0$ $x = -1$ $x = 3$ Therefore, $(-1, -\frac{1}{4}), (3, \frac{1}{4})$	<b>M1</b> <b>A2</b>
		<b>M1</b>

<b>7a</b>	<table border="1"> <tr> <td><b>x</b></td> <td>2</td> <td>4</td> <td>6</td> <td>8</td> </tr> <tr> <td><math>1 + 3\sqrt{x}</math></td> <td>5.243</td> <td>8.348</td> <td>8.348</td> <td>9.485</td> </tr> </table>	<b>x</b>	2	4	6	8	$1 + 3\sqrt{x}$	5.243	8.348	8.348	9.485	<b>M1</b> <b>A1</b>
<b>x</b>	2	4	6	8								
$1 + 3\sqrt{x}$	5.243	8.348	8.348	9.485								
	Area $\approx \frac{1}{2} \times 2 \times [5.243 + 9.485 + 2(7 + 8.348)]$	<b>B1</b> <b>M1</b> <b>A1</b>										
	$= 45.4$ (3 s.f)	<b>A1</b>										
<b>7b</b>	$\int_2^8 (1 + 3\sqrt{x}) dx$ $= [x + 2x^{\frac{3}{2}}]_2^8$ $[8 + 2(2\sqrt{2})^3] - [2 + 2(2\sqrt{2})]$ $= (8 + 32\sqrt{2}) - (2 + 4\sqrt{2})$ $= 6 + 28\sqrt{2}$	<b>M1</b> <b>A1</b> <b>M1</b> <b>M1</b> <b>A1</b>										
<b>7c</b>	$\frac{6+28\sqrt{2}-45.4}{6+28\sqrt{2}} \times 100\% = 0.43\%$	<b>M1</b> <b>A1</b>										

<b>8a</b>	$r = \frac{x+6}{x-2} = \frac{x^2}{x+6}$ $(x+6)^2 = x^2(x-2)$ $x^2 + 12x + 36 = x^3 - 2x^2$	<b>M1</b> <b>M1</b> <b>A1</b>
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	$x^3 - 3x^2 - 12x - 36 = 0$	
<b>8b</b>	When $x = 6$ , LHS = $216 - 108 - 72 - 36 = 0$ Therefore $x = 6$	<b>B1</b>
	$\begin{array}{r} x^2 + 3x + 6 \\ x-6 \overline{) x^3 - 3x^2 - 12x - 36} \\ \underline{x^3 - 6x^2} \phantom{- 36} \\ 3x^2 - 12x \phantom{- 36} \\ \underline{3x^2 - 18x} \phantom{- 36} \\ 6x - 36 \\ \underline{6x - 36} \\ 0 \end{array}$	<b>M1</b> <b>A1</b>
	$(x - 6)(x^2 + 3x + 6) = 0$ $x = 6$ $b^2 - 4ac = 3^2 - (4 \times 1 \times 6) = -15$	<b>M1</b> <b>A1</b>
	$b^2 - 4ac < 0$ Therefore, no real solutions to quadratic Therefore, no other solutions	<b>A1</b>
<b>8c</b>	$r = \frac{6+6}{6-2} = 3$	<b>B1</b>
<b>8d</b>	$a = 6 - 2 = 4$ $S_8 = \frac{4(3^8 - 1)}{301} = 13120$	<b>M1</b> <b>A1</b>

<b>9a</b>	$\frac{d}{dx}(\sec x) = \frac{d}{dx}[(\cos x)^{-1}]$ $= -(\cos x)^{-2} \times (-\sin x)$ $= \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x} \times \frac{\sin x}{\cos x}$ $= \sec x \tan x$	<b>M1</b> <b>A1</b> <b>M1</b> <b>A1</b>
<b>9b</b>	$\frac{dy}{dx} = 2e^{2x} \times \sec x + e^{2x} \times \sec x \tan x = e^{2x} \sec x (2 + \tan x)$ $x = 0$ $y = 1$ gradient = 2 $y = 2x + 1$	<b>M1</b> <b>A1</b> <b>M1</b> <b>A1</b>
<b>9c</b>	SP: $e^{2x} \sec x (2 + \tan x) = 0$ $\tan x = -2$ $x = -1.11$ (2 d.p)	<b>M1</b> <b>M1</b> <b>A1</b>

<b>10a</b>	$\frac{dy}{dx} = -e^2 x^{-2} + e^x$	<b>M1</b> <b>A1</b>
<b>10b</b>	SP: $-e^2 x^{-2} + e^x = 0$ Let $f(x) = -e^2 x^{-2} + e^x$ $f(1.3) = -0.70$ $f(1.4) = 0.29$ Sign change, $f(x)$ continuous therefore root.	<b>M1</b> <b>M1</b> <b>A1</b>
<b>10c</b>	$x = 2$ $y = \frac{3}{2}e^2$ Gradient = $\frac{3}{4}e^2$ $y - \frac{3}{2}e^2 = \frac{3}{4}e^2(x - 2)$ $y = \frac{3}{4}e^2 x$ $x = 0$ $y = 0$ So passes through origin	<b>M1</b> <b>M1</b> <b>A1</b> <b>A1</b>
<b>10d</b>	$x_1 = -1.125589$ $x_2 = -1.125803$	<b>M1</b> <b>A2</b>

	$x_3 = -1.125804$	
	$x$ – coordinate of $B = -1.1258$ (5 s.f)	<b>A1</b>
<b>11a</b>	$f(x) = (x - 1)^2 - 1 + 5 = (x - 1)^2 + 4$	<b>M1</b> <b>A1</b>
<b>11b</b>	$f(x) \geq 4$	<b>B1</b>
<b>11c</b>	$y = (x - 1)^2 + 4$ $(x - 1)^2 = y - 4$ $x - 1 = \pm \sqrt{y - 4}$ $x = \pm \sqrt{y - 4} + 1$	<b>M1</b>
	$f^{-1}(x) = 1 + \sqrt{x - 4}$	<b>M1</b> <b>A1</b>
<b>11d</b>	Translation by 4 units in negative $x$ direction Translation by 1 unit in negative $y$ direction (either first)	<b>B2</b>
<b>11e</b>	$\frac{dy}{dx} = \frac{1}{2}(x - 4)^{-\frac{1}{2}}$	<b>M1</b>
	$x = 8$ $y = 3$ gradient = $\frac{1}{4}$	<b>A1</b>
	Gradient of normal = - 4 $y - 3 = -4(x - 8)$ $y = 35 - 4x$	<b>M1</b> <b>A1</b>
<b>12</b>	$2 \sin y \cos y = \sin y$	<b>M1</b>
	$\sin y (2 \cos y - 1) = 0$	<b>M1</b>
	$\sin y = 0$ $\cos y = \frac{1}{2}$	<b>A1</b>
	$y = 0, \pi$ or $\frac{\pi}{3}, 2\pi - \frac{\pi}{3}$	<b>B1</b> <b>M1</b>
	$y = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$	<b>A2</b>

## Topic List

<b>Q1</b>	Implicit differentiation
<b>Q2</b>	Binomial expansion
<b>Q3</b>	Trig equations
<b>Q4</b>	Chain rule differentiation
<b>Q5</b>	Addition formulae
<b>Q6</b>	Factor theorem
<b>Q7</b>	Trapezium rule
<b>Q8</b>	Geometric series
<b>Q9</b>	Differentiation, stationary points
<b>Q10</b>	Stationary points, iteration
<b>Q11</b>	Differentiation, normal, completing the square
<b>Q12</b>	Solving trig equations

