Practice Exam Paper B
Time: 2 Hours

P1 P2

1a. Prove, by counter-example, that the statement "cosec $\theta - \sin\theta > 0$ for all values of θ in the interval $0 < \theta < \pi$" is false. **(2)**

b. Find the values of θ in the interval $0 < \theta < \pi$ such that cosec $\theta - \sin\theta = 2$, giving your answers to 2 decimal places. **(5)**

(Total Marks: 7)

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2. Solve each equation, giving your answers in exact form.

a. $\ln(2x - 3) = 1$ **(3)**

b. $3e^{y} + 5e^{-y} = 16$ **(5)**

(Total Marks: 8)

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3. A student completes a mathematics course and begins to work through past exam papers. He completes the first paper in 2 hours and the second in 1 hour 54 minutes.

Assuming that the times he takes to complete successive papers form a geometric sequence,

a. Find, to the nearest minute, how long he will take to complete the fifth paper **(3)**

b. Show that the total time he takes to complete the first eight papers is approximately 13 hours 28 minutes **(3)**

c. Find the least number of papers he must work through if he is to complete a paper in less than one hour. **(4)**

(Total Marks: 10)

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4. Given that $8\tan x - 3\cos x = 0$,

Show that

$$3\sin^2 x + 8\sin x - 3 = 0$$ **(3)**

b. Find, to 2 decimal places, the values of x in the interval $0 \leq x \leq 2\pi$ such that

$$8\tan x - 3\cos x = 0$$ **(5)**

(Total Marks: 8)

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5. The function f is defined by,

$$f(x) = 3 - x^2,\ x \geq 0$$

a. State the range of f **(1)**

b. Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same diagram **(3)**

c. Find an expression for $f^{-1}(x)$ and state its domain **(4)**

The function g is defined by, $g(x) \equiv \frac{8}{3-x}, x \neq 3$

d. Evaluate $fg(-3)$. **(2)**

e. Solve the equation $f^{-1}(x) = g(x)$. **(3)**

(Total Marks: 13)

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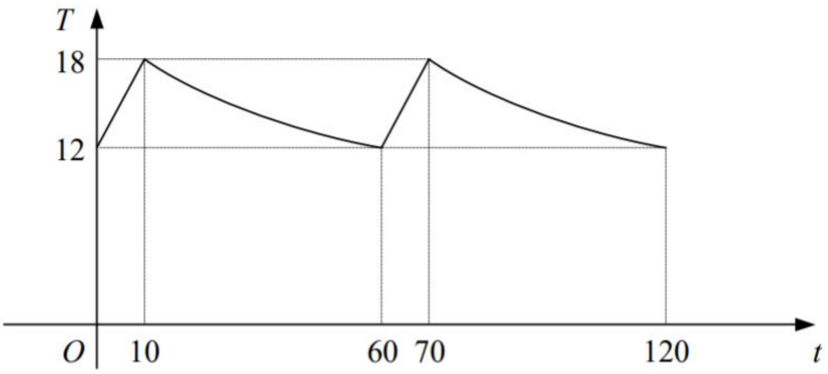
6. The figure shows a graph of the temperature of a room, T°C, at time t minutes.



The temperature is controlled by a thermostat such that when the temperature falls to 12°C, a heater is turned on until the temperature reaches 18°C. The room then cools until the temperature again falls to 12°C.

For t in the interval $10 \leq t \leq 60$, T is given by,

$$T = 5 + Ae^{-kt}$$

Where A and k are constants.

Given that $T = 18$ when $t = 10$ and that $T = 12$ when $t = 60$,

a. Show that $k = 0.0124$ to 3 significant figures and find the value of A **(6)**

b. Find the rate at which the temperature of the room is decreasing when $t = 20$ **(4)**

The temperature again reaches 18°C when $t = 70$ and the graph for $70 \leq t \leq 120$ is a translation of the graph for $10 \leq t \leq 60$

c. Find the value of the constant B such that for $70 \leq t \leq 120$

$$T = 5 + Be^{-kt}$$ **(3)**

(Total Marks: 13)

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7. A curve has the equation $y = x^2 - \sqrt{4 + \ln x}$

a. Show that the tangent to the curve at the point where $x = 1$ has the equation $7x - 4y = 11$ **(5)**

The curve has a stationary point with x-coordinate α.

b. Show that $0.3 < \alpha < 0.4$ **(3)**

c. Show that α is a solution of the equation,

$x = \frac{1}{2}(4 + \ln x)^{-\frac{1}{4}}$ **(2)**

d. Use the iteration formula,

$x_{n+1} = \frac{1}{2}(4 + \ln x_n)^{-\frac{1}{4}}$

$x_0 = 0.35$ to find x_1, x_2, x_3 and x_4, giving your answers to 5 decimal places. **(3)**

(Total Marks: 14)

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8. Find the binomial expansion of $(2 - 3x)^{-3}$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient. **(5)**

b. State the set of values of x for which your expansion is valid **(1)**

(Total Marks: 6)

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9. A curve has the equation

$$x^2 + 3xy - 2y^2 + 17 = 0.$$

a. Find an expression for $\frac{dy}{dx}$ in terms of x and y **(5)**

b. Find an equation for the normal to the curve at the point $(3, -2)$. **(3)**

(Total Marks: 8)

10a. Find the values of the constants A, B, C and D such that,

$\frac{2x^3 - 5x^2 + 6}{x^2 - 3x} \equiv Ax + B + \frac{C}{x} + \frac{D}{x-3}$ **(5)**

b. Evaluate,

$$\int_1^2 \frac{2x^3 - 5x^2 + 6}{x^2 - 3x}\, dx$$

Giving your answer in the form $p + q \ln 2$, where p and q are integers. **(5)**

(Total Marks: 10)

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11. A mathematician is selling goods at a car boot sale. She believes that the rate at which she makes sales depends on the length of time since the start of the sale, t hours, and the total value of sales she has made up to that time, £ x. She uses the model

$$\frac{dx}{dt} = \frac{k(5 - t)}{x}$$

Where k is a constant.

Given that after two hours she has made sales of £96 in total,

a. Solve the differential equation and show that she made £72 in the first hour of the sale. **(8)**

The mathematician believes that is it not worth staying at the sale once she is making sales at a rate of less than £10 per hour.

b. Verify that at 3 hours and 5 minutes after the start of the sale, she should have already left. **(4)**

(Total Marks: 12)

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12. A curve has parametric equations

$$x = 3\cos^2 t,\ y = \sin 2t \qquad 0 \leq t < \pi$$

a. Show that $\frac{dy}{dx} = -\frac{2}{3}\cot 2t$ **(4)**

b. Find the coordinates of the points where the tangent to the curve is parallel to the x-axis. **(3)**

c. Find the cartesian equations of the curve in the form $y^2 = f(x)$ **(4)**

(Total Marks: 11)

Total Marks: 120

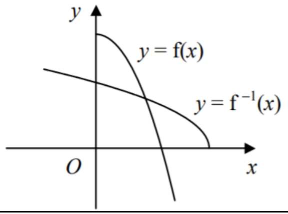
Mark Scheme

1a	If $\theta = \frac{\pi}{2}$, $\sin\theta = 1$ $\text{cosec}\,\theta = 1$	**M1**
	$\text{cosec}\,\theta - \sin\theta = 1 - 1 = 0$ Therefore, statement is false.	**A1**
1b	$1 - \sin^2\theta = 2\sin\theta$	**M1**
	$\sin^2\theta + 2\sin\theta - 1 = 0$ $\sin\theta = -1 - \sqrt{2}$ or $\sin\theta = -1 + \sqrt{2}$	**M1** **A1**
	$\theta = 0.4271$ $\theta = \pi - 0.4271$	**M1**
	$\theta = 0.4271, 2.71$	**A1**

2a	$2x - 3 = e$	**M1**
	$x = \frac{1}{2}(e + 3)$	**M1** **A1**
2b	$3e^{2y} - 16e^y + 5 = 0$	**M1**
	$(3e^y - 1)(e^y - 5) = 0$	**M1**
	$e^y = \frac{1}{3}$ $e^y = 5$	**A1**
	$y = \ln\frac{1}{3}$ $y = \ln 5$	**M1** **A1**

3a	$r = \frac{114}{12} = 0.95$	**M1**
	$u_5 = 120 \text{ x } (0.95)^4 = 97.74$	**M1**
	Therefore, 1 h 38 minutes	**A1**
3b	$S_8 = \frac{120\,[\,1-(0.95)^8]}{1-0.95}$	**M1** **A1**
	= 807.79 minutes = 13 hours 28 minutes	**A1**
3c	$120 \text{ x } (0.95)^{n-1} < 60$	**M1**
	$(n - 1)\log 95 < \log 0.5$	**M1**
	$n > \frac{\log 0.5}{\log 0.95} + 1$	**A1**
	$n > 14.51$, therefore 15 papers.	**A1**

4a	$\frac{8\sin x}{\cos x} - 3\cos x = 0$	**M1**
	$8\sin x - 3\cos^2 x = 0$ $8\sin x - 3(1 - \sin^2 x) = 0$	**M1**
	$3\sin^2 x + 8\sin x - 3 = 0$	**A1**
4b	$(3\sin x - 1)(\sin x + 3) = 0$	**M1**
	$\sin x = -3$ (no solutions) $\sin x = -\frac{1}{3}$	**A1**
	$x = 0.34$ $x = \pi - 0.34$	**B1** **M1**
	$x = 0.34$ $x = 2.80$	**A1**

5a	$f(x) \le 3$	**B1**
5b		**B3**
5c	$y = 3 - x^2$ $x^2 = 3 - y$	**M1**
	$x = \pm\sqrt{3-y}$ $f^{-1}(x) = \sqrt{3-x},\ x \le 3$	**M1** **A2**
5d	$f(\frac{4}{3}) = \frac{11}{9}$	**M1** **A1**
5e	$\sqrt{3-x} = \frac{8}{3-x}$ $3 - x = \frac{64}{(3-x)^2}$	**M1**
	$(3-x)^3 = 64$ $3 - x = 4$	**M1**
	$x = -1$	**A1**

6a	$t = 10$ $T = 18$ $18 = 5 + Ae^{-10k}$ $t = 60$ $T = 12$ $12 = 5 + Ae^{-60k}$	**M1**
	$A = \frac{13}{e^{-10k}} = 13e^{10k}$	**M1**
	$7 = 13e^{10k} \text{ x } e^{-60k}$ $e^{-50k} = \frac{7}{13}$	**A1**
	$k = -\frac{1}{50}\ln\frac{7}{13} = 0.0124$	**M1** **A1**
	$A = 13e^{10 \text{ x } 0.01238} = 14.7$	**A1**
6b	$T = 5 + 14.71e^{-0.01238t}$ $\frac{dT}{dt} = -0.01238 \times 14.71e^{-0.01238t} = -0.1822e^{-0.01238t}$	**M1** **A1**
	When $t = 20$, $\frac{dT}{dt} = -0.1822e^{-0.01238 \times 20} = -0.142$	**M1**
	Therefore, temperature decreasing at a rate of 0.142°C per minute (3 s.f)	**A1**
6c	$T = 5 + 14.71e^{-0.01238(t-60)}$	**M1**
	$= 5 + 14.71e^{0.7428 - 0.01238t}$ $= 5 + 14.71e^{0.7428} \text{ x } e^{-0.01238t}$	**M1**
	$= 5 + 30.9e^{-0.01238t}$ $B = 30.9$ (3 s.f)	**A1**

7a	$\frac{dy}{dx} = 2x - \frac{1}{2}(4 + lnx) \times \frac{1}{x} = 2x - \frac{1}{2x\sqrt{4+\ln x}}$	**M1** **A1**
	$x = 1$ $y = -1$ gradient $= \frac{7}{4}$	**A1**
	$y + 1 = \frac{7}{4}(x-1)$	**M1**
	$4y + 4 = 7x - 7$ $7x - 4y = 11$	**A1**

7b	SP: $2x - \frac{1}{2x\sqrt{4+\ln x}} = 0$	**M1**
	Let $f(x) = 2x - \frac{1}{2x\sqrt{4+\ln x}}$ $f(0.3) = -0.40$ $f(0.4) = 0.088$	**M1** **A1**
7c	$2x - \frac{1}{2x\sqrt{4+\ln x}} = 0$ $2x = \frac{1}{2x\sqrt{4+\ln x}}$ $x^2 = \frac{1}{4\sqrt{4+\ln x}} = \frac{1}{4}(4+\ln x)^{-\frac{1}{2}}$	**M1**
	$x = \sqrt{\frac{1}{4}(4+\ln x)^{-\frac{1}{2}}} = \frac{1}{2}(4+\ln x)^{-\frac{1}{4}}$	**A1**
	$x_1 = 0.38151$ $x_2 = 0.37877$ $x_3 = 0.37900$ $x_4 = 0.37898$	**M1** **A2**

8a	$= 2^{-3}(1 - \frac{3}{2}x)^{-3} = \frac{1}{8}(1 - \frac{3}{2}x)^{-3}$	**B1**
	$= \frac{1}{8}[1 + (-3)\left(-\frac{3}{2}x\right) + \frac{(-3)(-4)}{2}\left(-\frac{3}{2}x\right)^2 + \frac{(-3)(-4)(-5)}{3\times 2}(-\frac{3}{2}x)^3 + \ldots]$	**M1**
	$\frac{1}{8} + \frac{9}{16}x + \frac{27}{16}x^2 + \frac{135}{32}x^3 + \cdots$	**A3**
8b	$\lvert x \rvert < \frac{2}{3}$	**B1**

9a	$2x + 3y + 3x\frac{dy}{dx} - 4y\frac{dy}{dx} = 0$	**M1** **A2**
	$\frac{dy}{dx} = \frac{2x+3y}{4y-3x}$	**M1** **A1**
9b	Gradient $= \frac{6-6}{-8-9} = 0$	**M1**
	Normal parallel to y-axis $x = 3$	**M1** **A1**

10a	$2x^3 - 5x^2 + 6 \equiv (Ax + B)x(x-3) + C(x-3) + Dx$	**M1**
	$x = 0$ $6 = -3C$ $C = -2$ $x = 3$ $15 = 3D$ $D = 5$	**A1**
	Coefficients of x^3, $A = 2$	**B1**
	Coefficients of x^2, $B = 1$	**M1** **A1**
10b	$\int_1^2 2x + 1 - \frac{2}{x} + \frac{5}{x-3}\ dx$ $[x^2 + x - 2\ln\lvert x\rvert + 5\ln\lvert x-3\rvert]_1^2$	**M1** **A2**
	$(4 + 2 - 2\ln 2 + 0) - (1 + 1 + 0 + 5\ln 2)$	**M1**
	$= 4 - 7\ln 2$	**A1**

11a	$\int x\, dx = \int k(5-t)dt$	**M1**
	$\frac{1}{2}x^2 = k(5t - \frac{1}{2}t^2) + c$	**M1** **A1**
	$t = 0, x = 0$ $c = 0$	**B1**
	$t = 2, x = 96$	**M1**

	$4608 = 8k$ $k = 576$	**A1**
	$t = 1, \frac{1}{2}x^2 = 576 \times \frac{9}{2}$ $x = 72$	**M1** **A1**
11b	3 hours, 5 mins $t = 3.8033$ $x = 110.83$	**M1** **A1**
	$\frac{dx}{dt} = \frac{576(5-3.0833)}{110.83} = 9.96$ $\frac{dx}{dt} < 10$ so she should have left	**M1** **A1**

12a	$\frac{dx}{dt} = 6\cos t \times (-\sin t)$ $\frac{dy}{dx} = 2\cos 2t$	**M1** **A1**
	$\frac{dy}{dx} = \frac{2\cos 2t}{-6\cos t \sin t} = \frac{2\cos 2t}{-3\sin 2t} = -\frac{2}{3}\cot 2t$	**M1** **A1**
12b	$-\frac{2}{3}\cot 2t = 0$ $2t = \frac{\pi}{2}, t = \frac{\pi}{4}$ $2t = \frac{3\pi}{2}, t = \frac{3\pi}{4}$	**M1** **A1**
	$(\frac{3}{2}, 1)$ $(\frac{3}{2}, -1)$	**A1**
12c	$y^2 = \sin^2 2t = 4\sin^2 t\cos^2 t = 4(1-\cos^2 t)\cos^2 t$	**M2**
	$\cos^2 t = \frac{x}{3}$ $y^2 = 4(1-\frac{x}{3})\frac{x}{3}$ $y^2 = \frac{4}{9}x(3-x)$	**M1** **A1**

Topic List

Q1	Proof by contradiction and solving logs
Q2	Natural logs and exponentials
Q3	Sequences and series
Q4	Solving trig equations
Q5	Functions
Q6	Exponential modelling
Q7	Differentiation and numerical methods
Q8	Binomial expansion
Q9	Implicit differentiation
Q10	Partial fractions
Q11	Modelling differential equations
Q12	Parametric differentiation and cartesian equations