Practice Exam Paper A
Time: 2 Hours

P1 P2

1. Given that

$$x = \sec^2 y + \tan y,$$

show that $\frac{dy}{dx} = \frac{cos^2 y}{2\tan y + 1}$ **(4)**

(Total Marks: 4)

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2. Giving your answers to 2 decimal places, solve the simultaneous equations,

$$e^{2y} - x + 2 = 0$$
$$\ln(x + 3) - 2y - 1 = 0$$

(8)

(Total Marks: 8)

3a. Express $3 \cos x + \sin x$ in the form $R \cos (x - \alpha)^{\circ}$ where $R > 0$ and $0 < \alpha < 90^{\circ}$. **(4)**

b. Using your answer to part (a), or otherwise, solve the equations, $6 \cos^2 x + \sin 2x = 0$

for x in the interval, $0 \leq x \leq 360$, giving your answers to 1 decimal place where appropriate. **(6)**

(Total Marks: 10)

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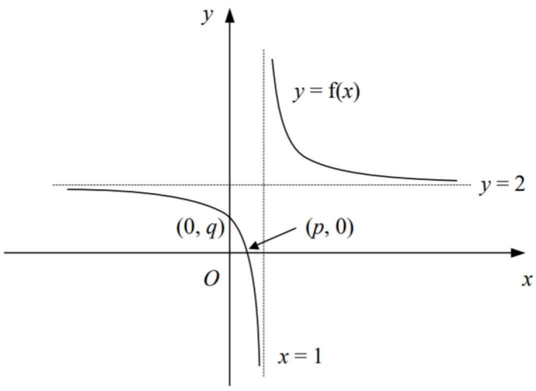
4. The figure shows the curve with equation $y = f(x)$. The curve crosses the axes at $(p, 0)$ and $(0, q)$ and the lines $x = 1$ and $y = 2$ are asymptotes of the curve.



a. Showing the coordinates of any points of intersection with the axes and the equations of any asymptotes, sketch on separate diagrams the graphs of,

i. $y = |f(x)|$
ii. $y = 2f(x + 1)$ **(6)**

Given also that, $f(x) \equiv \frac{2x-1}{x-1}$

b. Find the values of p and q **(3)**

c. Find an expression for $f^{-1}(x)$ **(3)**

(Total Marks: 12)

5. $f(x) = \frac{x^4+x^3-5x^2-9}{x^2+x-6}$

a. Using algebraic division, show that

$$f(x) = x^2 + A + \frac{B}{x+c}$$

where A, B and C are integers to be found. **(5)**

b. By sketching two suitable graphs on the same set of axes, show that the equation $f(x) = 0$ has exactly one real root. **(3)**

c. Use the iterative formula,

$$x_{n+1} = 2 + \frac{1}{{x_n}^2+1}$$

with a suitable starting value to find the root of the equation $f(x) = 0$ correct to 3 significant figures and justify the accuracy of your answer.

(5)

(Total Marks: 13)

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6. A curve has the equation $y = \sqrt{3x + 11}$
The point P on the curve has x-coordinate 3.

a. Show that the tangent to the curve at P has the equation,

$3x - 4\sqrt{5}y + 31 = 0$ **(6)**

The normal to the curve at P crosses the y-axis at Q.

b. Find the y-coordinate of Q in the form $k\sqrt{5}$ **(3)**

(Total Marks: 9)

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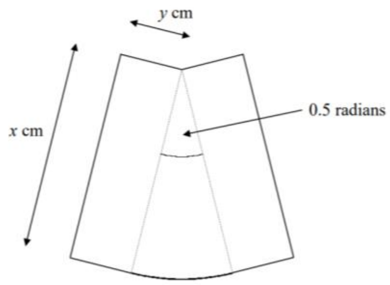
7. The figure shows a design consisting of two rectangles measuring x cm by y cm joined to a circular sector of radius x cm and angle 0.5 radians.



Given that the area of the design is 50 cm^2,

a. Show that the perimeter, P cm, of the design is given by, $P = 2x + \frac{100}{x}$ **(5)**

b. Find the value of x for which P is a minimum. **(4)**

c. Show that P is a minimum for this value of C. **(2)**

d. Find the minimum value of P in the form $k\sqrt{2}$. **(2)**

(Total Marks: 13)

8. A geometric series has first term a and common ratio r where $r > 1$. The sum of the first n terms of the series is denoted by Sn.

Given that $S_4 = 10 \times S_2$,

a. Find the value of r **(6)**

Given also that $S_3 = 26$,

b. Find the value of a, **(3)**

c. Show that $S_6 = 728$. **(2)**

(Total Marks: 11)

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9. Find the exact value of $\int_1^2 x \ln x \, dx$ **(6)**

(Total Marks: 6)

10a. Use the trapezium rule with two intervals of equal width to find an approximate value for the integral

$\int_0^2 arc \tan x \, dx$ **(5)**

b. Use the trapezium rule with four intervals of equal width to find an improved approximation for the value of the integral. **(2)**

(Total Marks: 7)

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11. Given that $y = -2$ when $x = 1$, solve the differential equation,

$$\frac{dy}{dx} = y^2 \sqrt{x}$$

Giving your answer in the form $y = f(x)$ **(7)**

(Total Marks: 7)

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12. $f(x) = \frac{15-17x}{(2+x)(1-3x)^2}, x \neq -2, x \neq \frac{1}{3}$

a. Find the values of the constants A, B and C such that,

$$f(x) = \frac{A}{2+x} + \frac{B}{1-3x} + \frac{C}{(1-3x)^2}$$

(4)

b. Find the value of $\int_{-1}^{0} f(x) dx$

giving your answer in the form $p + \ln q$, where p and q are integers. **(7)**

(Total Marks: 11)

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13. A curve has the equation $4x^2 - 2xy - y^2 + 11 = 0$

Find an equation for the normal to the curve at the point with the x-coordinate of -1. **(9)**

(Total Marks: 9)

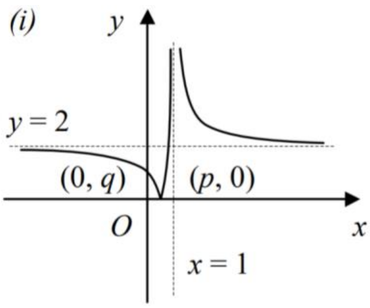
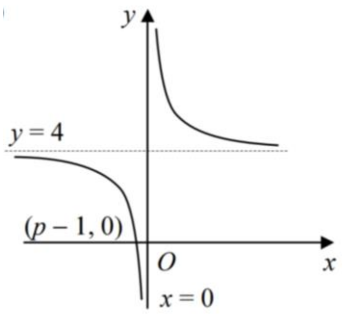
Total Marks: 120

<u>Mark Scheme</u>

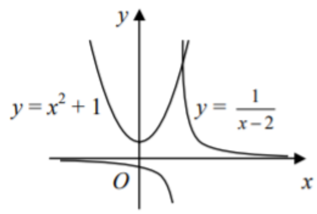
1	$\frac{dx}{dy} = 2\sec y \times \sec y \tan y + \ sec^2\ y = \ sec^2\ y(2\tan y + 1)$ $= \frac{2\tan y+1}{cos^2 y}$	**M1** **A1**
	$\frac{dy}{dx} = 1\ \div \frac{dx}{dy} = \frac{cos^2 y}{2\tan y+1}$	**M1** **A1**

2	$e^{2y} - x + 2 = 0$ $e^{2y} = x - 2$ $2y = \ln(x - 2)$	**M1**
	$\ln(x + 3) - \ln(x - 2) - 1 = 0$	**A1**
	$\ln \frac{x+3}{x+2} = 1$	**M1**
	$\frac{x+3}{x+2} = e$	**A1**
	$x + 3 = e(x - 2)$	**M1**
	$3 + 2e = x(e - 1)$	**M1**
	$x = \frac{2e+3}{e-1} = 4.91$ $y = \frac{1}{2}\ln\left(\frac{2x+3}{e-1} - 2\right) = 0.53$	**A2**

3a	$3\cos x + \sin x = R\cos x\cos\alpha$ $R\cos\alpha = 3$ $R\sin\alpha = 1$ $R = \sqrt{3^2 + 1^2} = \sqrt{10}$	**M1** **A1**
	$\tan\alpha = \frac{1}{3}$ $\alpha = 18.4^o$	**M1** **A1**
	$3\cos x + \sin x = \sqrt{10}\cos(x - 18.4)$	
3b	$6\cos^2 x + 2\sin x\cos x = 0$	**M1**
	$2\cos x(2\cos x + \sin x) = 0$	**M1**
	$\cos x = 0$ $3\cos x + \sin x = \sqrt{10}\cos(x - 18.4)$	**A1**
	$x = 90, 270$ $x - 18.4 = 90, 270$	**B1** **M1**
	$x = 90, 108.4, 270, 288.4$	**A1**

4ai		**M1** **A1**
4aii		**M2** **A2**

4b	$y = 0$ $2x - 1 = 0$ $x = \frac{1}{2}$ $p = \frac{1}{2}$	**M1** **A1**
	$x = 0$ $y = 1$ $q = 1$	**B1**
4c	$y = \frac{2x-1}{x-1}$ $y(x-1) = 2x - 1$	**M1**
	$x(y-2) = y - 1$ $x = \frac{y-1}{y-2}$	**M1**
	$f^{-1}(x) = \frac{x-1}{x-2}$	**A1**

5a	$\begin{array}{rrrrrr} & x^2 & +0x & +1 & & \\ x^2+x-6\,) & x^4 & +x^3 & -5x^2 & +0x & -9 \\ & x^4 & +x^3 & -6x^2 & & \\ & & & x^2 & +0x & -9 \\ & & & x^2 & +x & -6 \\ & & & & -x & -3 \end{array}$	**M1** **A1**
	$f(x) = x^2 + 1 + \frac{-x-3}{x^2+x-6}$	**A1**
	$= x^2 + 1 - \frac{x+3}{(x-2)(x+3)}$ $= x^2 + 1 - \frac{1}{x-2}$	**M1** **A1**
5b		**B2**
	$f(x) = 0$ $x^2 + 1 = \frac{1}{x-2}$ Graphs intersect onece, therefore exactly one real root	**B1**
5c	$x_0 = 3$ $x_1 = 2.1$ $x_2 = 2.1848$ $x_3 = 2.1732$ $x_4 = 2.1747$	**M1** **A1**
	Therefore root = 2.17 (3 s.f)	**A1**
	f(2.165) = -0.37 f(2.175) = 0.016	**M1**
	Sign change and $f(x)$ is continuous therefore root in interval.	**A1**

6a	$x = 3$ $y = \sqrt{20} = 2\sqrt{5}$	**B1**
	$\frac{dy}{dx} = \frac{1}{2}(3x+11)^{-\frac{1}{2}} \times 3 = \frac{3}{2}(3x+11)^{-\frac{1}{2}}$	**M1** **A1**
	Gradient $= \frac{3}{4\sqrt{5}}$	**A1**
	$y - 2\sqrt{5} = \frac{3}{4\sqrt{5}}(x-3)$	**M1**
	$4\sqrt{5}y - 40 = 3x - 9$ $3x - 4\sqrt{5}y + 31 = 0$	**A1**

6b	Normal: $y - 2\sqrt{5} = -\frac{4\sqrt{5}}{3}(x-3)$	**M1**
	at Q, $x = 0$ $y - 2\sqrt{5} = 4\sqrt{5}$ $y = 6\sqrt{5}$	**M1** **A1**

7a	$Area = 2xy + (\frac{1}{2} \times x^2 \times 0.5) = 2xy + \frac{1}{4}x^2 = 50$	**M1**
	$y = \frac{50 - \frac{1}{4}x^2}{2x} = \frac{25}{x} - \frac{1}{8}x$	**A1**
	$P = 2x + 4y + (x \text{ x } 0.5) = \frac{5}{2}x + 4y$	**M1**
	$= \frac{5}{2}x + 4\left(\frac{25}{x} - \frac{1}{8}x\right)$	**M1**
	$= \frac{5}{2}x + \frac{100}{x} - \frac{1}{2}x$ $= 2x + \frac{100}{x}$	**A1**
7b	$\frac{dP}{dx} = 2 - 100x^{-2}$	**M1** **A1**
	For minimum, $2 - 100x^{-2} = 0$ $x^2 = 50$	**M1**
	$x = \sqrt{50} = 5\sqrt{2}$	**A1**
7c	$\frac{d^2P}{dx^2} = 200x^{-3}$	**M1**
	$x = 5\sqrt{2}$ $\frac{d^2P}{dx^2} = \frac{2}{5}\sqrt{2}$ $\frac{d^2P}{dx^2} > 0$, therefore minimum	**A1**
7d	$2(5\sqrt{2}) + \frac{100}{5\sqrt{2}} = 10\sqrt{2} + 10\sqrt{2} = 20\sqrt{2}$	**M1** **A1**

8a	$\frac{a(r^4-1)}{r-1} = 10 \times \frac{a(r^2-1)}{r-1}$	**B1** **M1**
	$r^4 - 1 = 10(r^2 - 1)$ $r^4 - 10r^2 + 9 = 0$	**A1**
	$(r^2 - 1)(r^2 - 9) = 0$	**M1**
	$r^2 = 1, 9$ $r = \pm 1, \pm 3$	**M1**
	as $r > 1$, $r = 3$	**A1**
8b	$\frac{a(3^3-1)}{3-1} = 26$	**M1** **A1**
	$a = \frac{26}{13} = 2$	**A1**
8c	$S_6 = \frac{2(3^6-1)}{3-1} = 728$	**M1** **A1**

9	$u = \ln x$ $u' = \frac{1}{x}$ $v' = x$ $v = \frac{1}{2}x^2$	**M1**
	$I = [\frac{1}{2}x^2 \ln x]_1^2 - \int_1^2 \frac{1}{2}x \, dx$	**A1**
	$= [\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2]_1^2$	**M1** **A1**
	$= (2 \ln 2 - 1) - (0 - \frac{1}{4}) = 2 \ln 2 - \frac{3}{4}$	**M1** **A1**

10	see table below	**B2**

x	0	0.5	1	1.5	2
arc tan x	0	0.4636	0.7854	0..9829	1.1071

10a	$\frac{1}{2} \times 1 \times [0 + 1.1071 + 2(0.7854)] = 1.34$	**B1** **M1** **A1**
10b	$\frac{1}{2} \times 0.5 \times [0 + 1.1071 + 2(0.4636 + 0.7854 + 0.9828)] = 1.39$	**M1** **A1**

11	$\int \frac{1}{y^2}\, dy = \int \sqrt{x}\, dx$	**M1**
	$-y^{-1} = \frac{2}{3}x^{\frac{3}{2}} + c$	**M1** **A1**
	$x = 1$ $y = -2$ $\frac{1}{2} = \frac{2}{3} + c$ $c = -\frac{1}{6}$	**M1** **A1**
	$-\frac{1}{y} = \frac{2}{3}x^{\frac{3}{2}} - \frac{1}{6}$ $\frac{1}{y} = \frac{1}{6} - \frac{2}{3}x^{\frac{3}{2}}$ $= \frac{1}{6}\left(1 - 4x^{\frac{3}{2}}\right)$	**M1**
	$y = \frac{6}{1-4x^{\frac{3}{2}}}$	**A1**

12a	$15 - 17x \equiv A(1 - 3x)^2 + B(2 + x)(1 - 3x) + C(2 + x)$	
	$x = -2$ $49 = 49A$ $A = 1$	**B1**
	$x = \frac{1}{3}$ $\frac{28}{3} = \frac{7}{3}C$ $C = 4$	**B1**
	Coefficietns of x^2 $0 = 9A - 3B$ $B = 3$	**M1** **A1**
12b	$\int_{-1}^{0}(\frac{1}{2+x} + \frac{3}{1-3x} + \frac{4}{(1-3x)^2}\ dx$ $[\ln\lvert 2 + x\rvert - \ln\lvert 1 - 3x\rvert + \frac{4}{3}(1 - 3x)^{-1}]_{-1}^{0}$	**M1** **A3**
	$(\ln 2 + 0 + \frac{4}{3}) - (0 - \ln 4 + \frac{1}{3})$	**M1**
	$1 + \ln 8$	**M1** **A1**

13	When $x = -1$ $y = -3$	**A1**
	$8x - 2y - 2x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$	**M1** **A2**
	At (-1, -3) $-8 + 6 + 2\frac{dy}{dx} + 6\frac{dy}{dx} = 0$ $\frac{dy}{dx} = \frac{1}{4}$	**M1** **A1**
	Gradient of normal = -4	**M1**
	$y + 3 = -4(x + 1)$ $y = -4x - 7$	**M1** **A1**

Topic List

Q1	Trig differentiation
Q2	Solving simultaneous equations with logs and exponentials
Q3	Trigonometry
Q4	Functions
Q5	Algebraic division, graph sketching and iteration
Q6	Equations of normal
Q7	Sectors, maximum/minimum problem
Q8	Geometric series
Q9	Definite integrals using integration by parts
Q10	Trapezium rule
Q11	Integration by separating variables
Q12	Partial fractions
Q13	Implicit differentiation