Numerical Methods
Pt. 2: Newton–Raphson Method

A-Level
Pt.1 : Iteration
Pt. 2: Newton-Raphson Method

1. The equation $2x^3 + x^2 - 1 = 0$ has exactly one real root.

a. Find the Newton-Raphson formula for this equations. **(3)**

Using your formula from part a with $x_1 = 1$,

b. Find the values of x_2 and x_3. **(2)**

c. Explain why, for this question, the Newton-Raphson method cannot be used with $x_1 = 0$. **(1)**

2. $f(x) = 5x^2 - 4x^{\frac{3}{2}} - 6, x \geq 0$
The root α of the equation $f(x) = 0$ lies in the interval [1.6, 1.8].

a. Use linear interpolationonce on the interval [1.6, 1.8] to find an approximation to α. Give your answer to 3 decimal places. **(4)**

b. Differentiate $f(x)$ to find $f'(x)$. **(2)**

c. Taking 1.7 as a first approximation to α, apply the Newton-Raphson process once to $f(x)$ to obtain a second approximation to α. Give your answer to 3 decimal places. **(4)**

3a. Show that $f(x) = x^4 + x - 1$ has a real root α in the interval [0.5, 1.0]. **(2)**

b. Starting with the interval [0.5, 1.0], use interval bisection twice to find an interval of width 0.125 which contains α. **(3)**

c. Taking 0.75 as a first approximation, apply the Newton-Raphson process twice to $f(x)$ to obtain an approximate value of α. Give your answer to 3 decimal places. **(4)**

Mark Scheme

1a.

let f(x) = $2x^3 + x^2 - 1$ f'(x) = $6x^2 + 2x$	**M1**
$x_{n+1} = x_n - \frac{2{x_n}^3 + {x_n}^2 - 1}{6{x_n}^2 + 2x_n}$ $= \frac{x_n(6{x_n}^2 + 2x_n) - (2{x_n}^3 + {x_n}^2 - 1)}{6{x_n}^2 + 2x_n}$	**M1**
$= \frac{4{x_n}^3 + {x_n}^2 + 1}{6{x_n}^2 + 2x_n}$	**M1**

1b.

$x_2 = \frac{4(1)^3 + (1)^2 + 1}{6(1)^2 + 2(1)} = \frac{6}{8} = \frac{3}{4}$	**M1**
$x_3 = \frac{4(\frac{3}{4})^3 + (\frac{3}{4})^2 + 1}{6(\frac{3}{4})^2 + 2(\frac{3}{4})} = \frac{\frac{13}{4}}{\frac{39}{8}} = \frac{2}{3}$	**M1**

1c.

$x_{n+1} = \frac{4{x_n}^3 + {x_n}^2 + 1}{6{x_n}^2 + 2x_n}$	**M1**
$x_3 = \frac{4(\frac{3}{4})^3 + (\frac{3}{4})^2 + 1}{6(\frac{3}{4})^2 + 2(\frac{3}{4})} = \frac{\frac{13}{4}}{\frac{39}{8}} = \frac{2}{3}$	**M1**

2a.

f(1.6) = -1.295430 …. f(1.8) = 0.540186…	**M1**
$\frac{\alpha - 1.6}{1.8 - \alpha} = \frac{1.295430}{0.540186}$	**M1**
$\alpha - 1.6 = (2.39811\ldots.)(1.8 - \alpha)$	**M1**
$\alpha = 1.741$	**M1**

2b.

f'(x) = $10x - 6x^{\frac{1}{2}}$	**M1 M1**

2c.

$x_1 = 1.7$ $x_2 = 1.7 - \frac{f(1.7)}{f'(1.7)}$	**M1**
$x_2 = 1.7 - \frac{5(1.7)^2 - 4(1.7)^{\frac{3}{2}} - 6}{10(1.7) - 6(1.7)^{\frac{1}{2}}}$	**M1 M1**
$x_2 = 1.7 - \frac{-0.4161527\ldots}{9.176957\ldots}$	**M1**
$x_2 = 1.74534\ldots = 1.745$ (3.dp)	**M1**

3a.

f(0.5) = -0.4375 f(1.0) = 1	**M1**
A root lies in the interval since there is a change in sign and f(x) is continuous over this interval.	**M1**

3b.

0.75 - 0.125 = 0.625	**M1**
f(0.75) = 0.06640	**M1**
f(0.625) = -0.2224	**M1**

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3c.

$f(x) = x^4 + x - 1$ $f'(x) = 4x^3 + 1$	**M1**
$x_1 = 0.75$ $x_2 = 0.75 - \frac{(0.75)^4 + (0.75) - 1}{4(0.75)^3 + 1}$	**M1**
$x_2 = 0.7525290\ldots = 0.725$ (to 3 d.p)	**M1**
$x_3 = 0.72449\ldots = 0.724$ (to 3 d.p)	**M1**