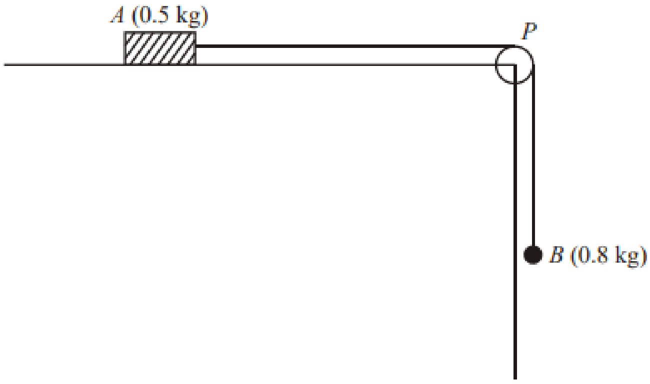
A2 Mechanics Practice Paper D
60 Marks

60
Minutes

1.



A block of wood A of mass 0.5 kg rests on a rough horizontal table and is attached to one end of a light inextensible string. The string passes over a small smooth pulley P fixed at the edge of the table. The other end of the string is attached to a ball B of mass 0.8 kg which hangs freely below the pulley, as shown in the diagram above. The coefficient of friction between A and the table is μ. The system is released from rest with the string taut. After release, B descends a distance of 0.4 m in 0.5 s. Modelling A and B as particles, calculate,

a. The acceleration of B, **(3)**

b. The tension in the string, **(4)**

c. The value of μ. **(5)**

d. State how in your calculations you have used the information that the string is inextensible. **(1)**

(Total: 13 marks)

2. A train is travelling at 10 ms^{-1} on a straight horizontal track. The driver sees a red signal 135 m ahead and immediately applies the brakes. The train immediately decelerates with constant deceleration for 12 s, reducing its speed to 3 ms^{-1}. The driver then releases the brakes and allows the train to travel at a constant speed of 3 ms^{-1} for a further 15s. He then applies the brakes again and the train slows down with constant deceleration, coming to rest as it reaches the signal.

a. Sketch a speed-time graph to show the motion of the train. **(3)**

b. Find the distance travelled by the train from the moment when the brakes are first applied to the moment when its speed first reaches 3 ms^{-1}. **(2)**

c. Find the total time from the moment when the brakes are first applied to the moment when the train comes to rest.

(5)

(Total: 10 marks)

3. A box of emergency supplies is dropped to victims of a natural disaster from a stationary helicopter at a height of 1000 metres. The initial velocity of the box is zero. At time t s after being dropped, the acceleration, a m s^{-2}, of the box in the vertically downwards direction is modelled by,

$$a = 10 - t \text{ for } 0 \leq t \leq 10,$$
$$a = 0 \text{ for } t > 10.$$

a. Find an expression for the velocity, v m s^{-1}, of the box in the vertically downwards direction in terms of t for $0 \leq t \leq 10$. Show that for $t > 10$, $v = 50$.

(4)

b. Draw a sketch graph of v against t for $0 \leq t \leq 20$. **(3)**

c.

i. Show that the height, h m, of the box above the ground at time t s is given, for $0 \leq t \leq 10$, by

$$h = 1000 - 5t^2 + \frac{1}{6}t^3.$$

ii. Find the height of the box when $t = 10$. **(4)**

d. Find the value of t when the box hits the ground. **(2)**

e. Some of the supplies in the box are damaged when the box hits the ground. So measures are considered to reduce the speed with which the box hits the ground the next time one is dropped. Two different proposals are made. Carry out suitable calculations and then comment on each of them.

(A) The box should be dropped from a height of 500 m instead of 1000 m.

(2)

(B) The box should be fitted with a parachute so that its acceleration is given by:

$$a = 10 - 2t \text{ for } 0 \leq t \leq 5,$$
$$a = 0 \text{ for } t > 5.$$

(3)

(Total: 18 marks)

4. One end of a light inextensible string is attached to a block P of mass 5 kg. The block P is held at rest on a smooth fixed plane which is inclined to the horizontal at an angle α, where $\sin \alpha = \frac{3}{5}$.

The string lies along a line of greatest slope of the plane and passes over a smooth light pulley which is fixed at the top of the plane.

The other end of the string is attached to a light scale pan which carries two blocks Q and R, with block Q on top of block R.

The mass of block Q is 5 kg and the mass of block R is 10 kg.

The scale pan hangs at rest and the system is released from rest. By modelling the blocks as particles, ignoring air resistance and assuming the motion is uninterrupted, find,

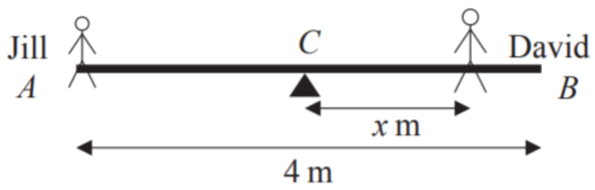
a. (i) The acceleration of the scale pan,
(ii) The tension in the string, **(8)**

b. The magnitude of the force exerted on block Q by block R, **(3)**

c. The magnitude of the force exerted on the pulley by the string. **(5)**

(Total: 16 marks)

5. A seesaw in a playground consists of a beam AB of length 4 m which is supported by a smooth pivot at its centre C. Jill has mass 25 kg and sits on the end A. David has mass 40 kg and sits at a distance x metres from C, as shown in the figure.



The beam is initially modelled as a uniform rod. Using this model,

a. Find the value of x for which the seesaw can rest in equilibrium in a horizontal position. **(3)**

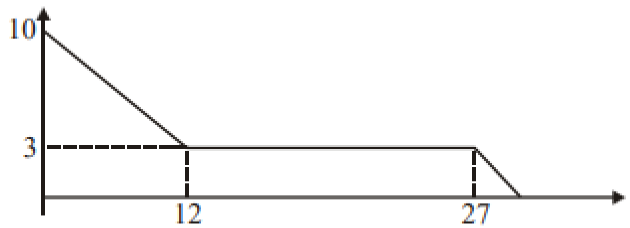
b. State what is implied by the modelling assumption that the beam is uniform. **(1)**

(Total: 4 marks)

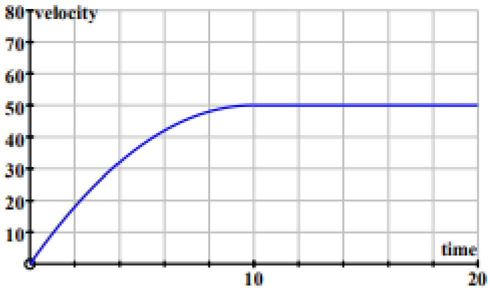
Total Marks: 60

Mark Scheme

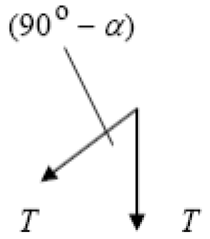
1a	'$s = ut + \frac{1}{2}at^2$' for B: $0.4 = \frac{1}{2}a(0.5)^2$ $a = \underline{3.2 \text{ m s}^{-2}}$	M1 A1 A1
1b	N2L for B: $0.8g - T = 0.8 \times 3.2$ $T = \underline{5.28 \text{ or } 5.3 \text{ N}}$	M1 A1ft ↓ M1 A1
1c	A: $F = \mu \times 0.5g$ N2L for A: $T - F = 0.5a$ Sub and solve $\mu = \underline{0.75 \text{ or } 0.751}$	B1 M1 A1 ↓ M1 A1
1d	Same acceleration for A and B.	B1

2a	 Shape $0 < t < 12$ Shape $t > 12$ Figures	B1 B1 B1
2b	Distance in 1st 12 s = $\frac{1}{2} \times (10 + 3) \times 12$ *or* $(3 \times 12) + \frac{1}{2} \times 3 \times 7$ = $\underline{78 \text{ m}}$	M1 A1
2c	**either** distance from $t = 12$ to $t = 27 = 15 \times 3 = 45$ $\therefore$ distance in last section = $135 - 45 = 12$ m $\frac{1}{2} \times 3 \times t = 12$, $\Rightarrow t = 8$ s hence total time = $27 + 8 = \underline{35 \text{ s}}$ Distance remaining after 12 s = $135 - 78 = 57$ m $\frac{1}{2} \times (15 + 15 + t) \times 3 = 57$ $\Rightarrow t = 8$ Hence total time = $27 + 8 = \underline{35 \text{ s}}$	B1ft M1 A1ft A1 A1 B1ft M1 A1ft A1 A1

3a	Integrate a to obtain v	Attempt to integrate	**M1**
	$v = 10t - \frac{1}{2}t^2 \quad (+c)$		**A1**
	$t = 10 \Rightarrow v = 100 - 50 = 50$	Substitution of $t = 10$ to find v	**M1**
	Since $a = 0$ for $t > 10$, $v = 50$ for $t > 10$	Sound argument required for given answer. It must in some way refer to $a = 0$.	**A1**

3b	Continuous two part v-t graph	The graph must cover $t = 0$ to $t = 20$	**B1**
			
	Curve for $0 \le t \le 10$		**B1**
	Horizontal straight line for $10 \le t \le 20$	B0 if no vertical scale is given	**B1**
3c	Distance fallen = $\int \left(10t - \frac{1}{2}t^2\right) dt$	Attempt to integrate	**M1**
	$d = 5t^2 - \frac{1}{6}t^3 + c \quad (c = 0)$		**A1**
	Height = $1000 - d$		
	Height = $1000 - 5t^2 + \frac{1}{6}t^3$	This mark should only be given if the signs are correctly obtained.	**A1**
	When $t = 10$, $h = 667$	oe	**B1**
3d	Time at constant vel $= 667 \div 50 = 13.3$	FT for h from part (iii)	**B1**
	Total time $t = 10 + 13.3 = 23.3$	FT	**B1**
3e	*A* Since $500 > 333$	For finding the height at which the crate reaches terminal velocity, eg $h = 16$ equivalent relevant calculation. FT for h from part (iii) if used.	**M1**
	The box will have reached terminal speed. So there is no improvement	Allow either one (or both) of these two statements.	**A1**
	B $v = 10t - t^2 \quad (\text{for } t \le 5)$	Integration to find v	**M1**
	Terminal velocity is 25 m s^{-1}		**A1**
	So better		**A1**

4a	$T - 5g\sin\alpha = 5a$	M1 A1
	$15g - T = 15a$	M1 A1
	solving for a	M1
	$a = 0.6g$	A1
	solving for T	M1
	$T = 6g$	A1
4b	For Q: $5g - N = 5a$ $N = 2g$	M1 A1 A1 f.t.

4c	 $F = 2T\cos(\frac{90° - \alpha}{2})$ $= 12\text{g}\cos 26.56..°$ $= 105$ N	M1 A2 A1 f.t. A1

5a	M(C): 25g x 2 = 40g x x	**M1** **A1**
	$x = 1.25m$b	**A1**
5b	Weight/mass acts at a mid-point Or, weight/mass evenly distributed	**B1**