A2 Mechanics Practice Paper B
60 Marks

60 Minutes

1. At time $t = 0$, a football player kicks a ball from the point *A* with position vector $(2\mathbf{i} + \mathbf{j})$ m on a horizontal football field. The motion of the ball is modelled as that of a particle moving horizontally with constant velocity $(5\mathbf{i} + 8\mathbf{j})$ m s^{-1}. Find

a. The speed of the ball, **(2)**

b. The position vector of the ball after *t* seconds. **(2)**

The point *B* on the field has position vector $(10\mathbf{i} + 7\mathbf{j})$ m.

c. Find the time when the ball is due north of *B*. **(2)**

At time $t = 0$, another player starts running due north from *B* and moves with constant speed *v* m s^{-1}. Given that he intercepts the ball,

d. Find the value of *v*. **(6)**

e. State one physical factor, other than air resistance, which would be needed in a refinement of the model of the ball's motion to make the model more realistic. **(1)**

(Total: 13 marks)

2. At time *t* = 0, a particle is projected vertically upwards with speed *u* ms^{-1} from a point 10 m above the ground. At time *T* seconds, the particle hits the ground with speed 17.5 ms^{-1}. Find

a. The value of *u*, **(3)**

b. The value of *T*. **(4)**

(Total: 7 marks)

3. A particle *P* moves on the *x*-axis. The acceleration of *P* at time *t* seconds is $(4t - 8)$ ms^{-2}, measured in the direction of *x* increasing. The velocity of *P* at time *t* seconds is *v* m s^{-1}. Given that *v* = 6 when *t* = 0, find

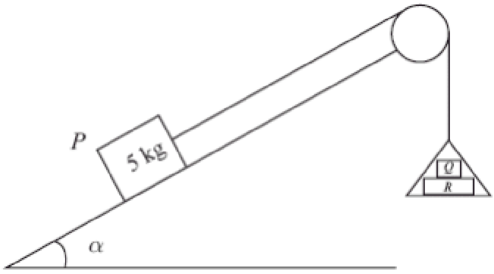
a. *v* in terms of *t*, **(4)**

b. The distance between the two points where *P* is instantaneously at rest. **(7)**

(Total: 11 marks)

Please turn over

4.



A car of mass 800 kg pulls a trailer of mass 200 kg along a straight horizontal road using a light towbar which is parallel to the road. The horizontal resistances to motion of the car and the trailer have magnitudes 400 N and 200 N respectively. The engine of the car produces a constant horizontal driving forced on the car of magnitude 1200 N. Find

a. The acceleration of the car and trailer, **(3)**

b. The magnitude of the tension in the towbar **(3)**

c. The car is moving along the road when the driver sees a hazard ahead. He reduces the force produced by the engine to zero and applies the brakes. The brakes produce a force on the car of magnitude F newtons and the car and trailer decelerate. Given that the resistances to motion are unchanged and the magnitude of the thrust in the towbar is 100 N, find the value of F. **(7)**

(Total: 13 marks)

5. A uniform rod AB has length 1.5 m and mass 8 kg. A particle of mass m kg is attached to the rod at B. The rod is supported at the point C, where AC = 0.9 m, and the system is in equilibrium with AB horizontal, as shown in the diagram above.

a. Show that $m = 2$. **(4)**

A particle of mass 5 kg is now attached to the rod at A and the support is moved from C to a point D of the rod. The system, including both particles, is again in equilibrium with AB horizontal.

b. Find the distance AD **(5)**

(Total: 9 marks)

6. A particle P of mass 2 kg is moving under the action of a constant force F newtons. When $t = 0$, P has velocity $(3\mathbf{i} + 2\mathbf{j})$ m s^{-1} and at time $t = 4$ s, P has velocity $(15\mathbf{i} - 4\mathbf{j})$ m s^{-1}. Find

(a) the acceleration of P in terms of **i** and **j**, **(2)**

(b) the magnitude of F, **(4)**

(Total: 6 marks)

Total Marks: 60

Mark Scheme

1a	Speed of ball = $\sqrt{(5^2 + 8^2)} \approx$ <u>9.43 m s^{-1}</u> *M1 Valid attempt at speed (square, add and squ. root cpts)*	M1 A1
1b	p.v. of ball = $(2\mathbf{i} + \mathbf{j}) + (5\mathbf{i} + 8\mathbf{j})t$ *M1 needs non-zero p.v. + (attempt at veloc vector) x t. Must be vector*	M1 A1
1c	North of B when $\mathbf{i}$ components same, i.e. $2 + 5t = 10$ $t =$ <u>1.6 s</u>	M1 A1
1d	When $t = 1.6$, p.v. of ball = $10\mathbf{i} + 13.8\mathbf{j}$ (or $\mathbf{j}$ component = 13.8) Distance travelled by 2nd player = 13.8 – 6 = 6.8 Speed = 6.8 ÷ 1.6 = <u>4.25 m s^{-1}</u> *or* $[(2 + 5t)\mathbf{i} +]\ (1 + 8t)\mathbf{j} = [10\mathbf{i} +]\ (7 + vt)\mathbf{j}$ *(pv's or* $\mathbf{j}$ *components same)* Using $t = 1.6$: $1 + 12.8 = 7 + 1.6v$ (equn in v only) $v =$ <u>4.25 m s^{-1}</u> *2nd M1 – allow if finding displacement vector (e.g. if using wrong time)* *3rd M1 for getting speed as a scalar (and final answer must be as a scalar). But if they get e.g. '4.25j', allow M1 A0*	M1A1 ↓ M1A1 ↓ M1A1 M1A1 ↓ M1A1 ↓ M1A1
1e	Allow for friction on field (i.e. velocity of ball not constant) *or* allow for vertical component of motion of ball *Allow 'wind', 'spin', 'time for player to accelerate', size of ball* *Do not allow on their own 'swerve', 'weight of ball'.*	B1

2a	$v^2 = u^2 + 2as \Rightarrow 17.5^2 = u^2 + 2 \times 9.8 \times 10$ Leading to $u = 10.5$	M1 A1 A1
2b	$v = u + at \Rightarrow 17.5 = -10.5 + 9.8T$ $T = 2\frac{6}{7}$ (s) Alternatives $s = \left(\frac{u+v}{2}\right)T \Rightarrow 10 = \left(\frac{17.5 + -10.5}{2}\right)T$ $\frac{20}{7} = T$ $s = ut + \frac{1}{2}at^2 \Rightarrow -10 = 10.5t - 4.9t^2$ Leading to $T = 2\frac{6}{7}, \left(-\frac{5}{7}\right)$ Rejecting negative (b) can be done independently of (a) $s = vt - \frac{1}{2}at^2 \Rightarrow -10 = -17.5t + 4.9t^2$ Leading to $T = 2\frac{6}{7}, \frac{5}{7}$ For final A1, second solution has to be rejected. $\frac{5}{7}$ leads to a negative u.	M1A1ft DM1A1 M1A1ft DM1A1 M1A1ft DM1A1 M1A1 DM1 A1

3a	$v = \int a\,dt = 2t^2 - 8t\ (+c)$ Using $v = 6$, $t = 0$; $v = 2t^2 - 8t + 6$	M1 A1 M1 A1
3b	$v = 0 \Rightarrow 2t^2 - 8t + 6 = 0, \Rightarrow t = 1,3$ $S = \int(2t^2 - 8t + 6)\mathrm{d}t = \left[\frac{2}{3}t^3 - 4t^2 + 6t\right]$ $= 0 - 2\frac{2}{3}$ Distance is $(\pm)2\frac{2}{3}$ m	M1 A1 M1 A2, 1, 0 M1 A1

4a	For whole system: $1200 - 400 - 200 = 1000a$ $a = 0.6 \text{ m s}^{-2}$	M1 A1 A1
4b	For trailer: $T - 200 = 200 \times 0.6$ $T = 320$ N **OR:** For car: $1200 - 400 - T = 800 \times 0.6$ $T = 320$ N	M1 A1 **ft** A1 **OR:** M1 A1 **ft** A1
4c	For trailer: $200 + 100 = 200f$ or $-200f$ $f = 1.5 \text{ m s}^{-2}$ (-1.5) For car: $400 + F - 100 = 800f$ or $-800f$ $F = 900$ (N.B. For both: $400 + 200 + F = 1000f$)	M1 A1 A1 M1 A2 A1

5a	M(C)	**M1**
	8g x (0.9 – 0.75)	**A1**
	$= mg(1.5 - 0.9)$	**M1**
	Therefore $m = 2$	**A1**
5b	M(D)	**M1**
	$5g \times x = 8g \times (0.75 - x) + 2g(1.5 - x)$	**M1** **A1**
	Solving, $x = 0.6$	**M1** **A1**

6a	$a = \frac{(15i - 4j) - (3i + 2j)}{4}$	**M1**
	$= 3\mathbf{i} - 15\mathbf{j}$	**A1**
6b	$F = ma$	**M1**
	$F = 6\mathbf{i} - 3\mathbf{j}$	**A1**
	$\lvert F \rvert = \sqrt{(6^2 + 3^2)}$	**M1**
	= 6.71 N	**A1**