A-level Further Mathematics Further Pure 1 and 2
Paper D

1a. Find the four roots of the equation,

$$z^4 = 8(\sqrt{3} + \mathrm{i})$$

in the form $z = re^{i\theta}$ **[5]**

b. Show these roots on an Argand diagram **[2]**

(Total 7 marks)

2. The points $P(4k^2, 8k)$ and $Q(k^2, 4k)$, where k is a constant, lie on parabola C with equation $y^2 = 16x$

The straight line l_1 passes through the points P and Q.

a. Show that an equation of the line l_1 is given by,

$$3ky - 4x = 8k^2$$

[4]

The line l_2 is perpendicular to the line l_1 and passes through the focus of the parabola C. The line l_2 meets the directrix of C at the point R.

b. Find, in terms of k, the y coordinates of the point R.

[7]

(Total 11 marks)

3a. Express $\frac{3}{(3r-1)(3r+2)}$ in partial fractions. **[2]**

b. Using your answer to part (a) and the method of differences, show that,

$$\sum_{r=1}^{n} \frac{3}{(3r-1)(3r+2)} = \frac{3n}{2(3n+2)}$$

[3]

c. Evaluate $\sum_{r=100}^{1000} \frac{3}{(3r-1)(3r+2)}$, giving your answer to 3 significant figures. **[2]**

(Total 7 marks)

4a. Given that $y = x\, arc \sin x\,, \quad 0 \leq x \leq 1$

i. an expression for $\frac{dy}{dx}$

ii. The exact value of $\frac{dy}{dx}$ when $x = \frac{1}{2}$

b. Given that $y = arc \tan(3e^{2x})$, show that,

$$\frac{dy}{dx} = \frac{3}{5\cosh 2x + 4\sinh 2x}$$

(Total 8 marks)

5. Show that,

a. $\int_5^8 \frac{1}{x^2-10x+34}\,dx = k\pi$, giving the values of the fraction k **[5]**

b. $\int_5^8 \frac{1}{\sqrt{x^2-10x+34}}\,dx = \ln(A+\sqrt{n})$, giving the values of the integers A and n **[4]**

(Total 9 marks)

6a. Show that the transformation $z = y^{\frac{1}{2}}$ transforms the differential equation.

$$\frac{dy}{dx} - 4y\tan x = 2y^{\frac{1}{2}} \qquad \text{(I)}$$

Into the differential equation,

$$\frac{dz}{dx} - 2z\tan x = 1 \qquad \text{(II)}$$

[5]

b. Sole the differential equation (II) to find z as a function of x **[6]**

c. Hence obtain the general solution of the differential equation (I) **[1]**

(Total 12 marks)

7. Find the set of values of x for which,

$$x + 4 > \frac{2}{x+3}$$

[6]

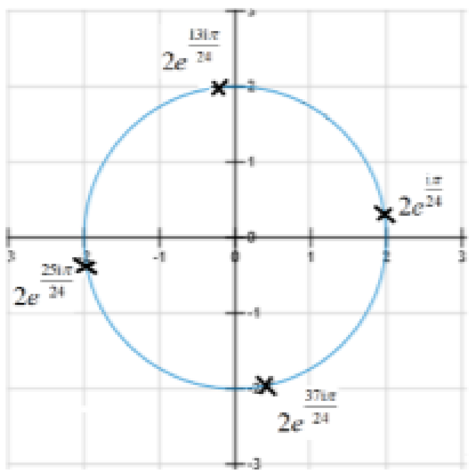
(Total 6 marks)

End of Paper

Total Marks: 60

Mark Scheme

Question 1

Question Number	Scheme	Notes	Marks
3	$z^4 = 8(\sqrt{3} + \text{i})$		
(a)	$\left(\lvert z^4 \rvert = \sqrt{(8\sqrt{3})^2 + 8^2} = \sqrt{256} = \right)$ 16 or $(\lvert z \rvert =)$ 2	Give B1 for either 16 or 2 seen anywhere	B1
	$(\arg z =) \arctan \dfrac{1}{\sqrt{3}} = \dfrac{\pi}{6}$	$\dfrac{\pi}{6}$ Accept 0.524	B1
	$r^4 = 16 \Rightarrow r = 2$		
	$4\theta = -\dfrac{23\pi}{6}, -\dfrac{11\pi}{6}, \dfrac{\pi}{6}, \dfrac{13\pi}{6}$	Range not specified, you may see $4\theta = \dfrac{\pi}{6}, \dfrac{13\pi}{6}, \dfrac{25\pi}{6}, \dfrac{37\pi}{6}$	
	$\theta = -\dfrac{23\pi}{24}, -\dfrac{11\pi}{24}, \dfrac{\pi}{24}, \dfrac{13\pi}{24}$	Clear attempt at both r and θ with at least 2 different values for their $\arg z$, ie $r = \sqrt[4]{\text{their } 16}$, $\theta = \dfrac{\text{principal arg} + 2n\pi}{4}$ all 4 correct distinct values of θ cao. $\theta = \dfrac{\pi}{24}, \dfrac{13\pi}{24}, \dfrac{25\pi}{24}, \dfrac{37\pi}{24}$ scores A1	M1, A1
	Roots are		
	$2\text{e}^{\frac{-23\text{i}\pi}{24}}, 2\text{e}^{\frac{-11\text{i}\pi}{24}}, 2\text{e}^{\frac{\text{i}\pi}{24}}, 2\text{e}^{\frac{13\text{i}\pi}{24}}$	All in correct form cao $2\text{e}^{\frac{\text{i}\pi}{24}}, 2\text{e}^{\frac{13\text{i}\pi}{24}}, 2\text{e}^{\frac{25\text{i}\pi}{24}}, 2\text{e}^{\frac{37\text{i}\pi}{24}}$ scores A1	A1
			(5)
(b)		B1: All 4 radius vectors to be the same length (approx) and perpendicular to each other. Circle not needed. Radius vector lines need not be drawn. If lines drawn and marked as perpendicular, accept for B1 B1: All in correct position relative to axes. Points marked must be close to the relevant axes. At least one point to be labelled or indication of scale given.	B1B1
			(2)
			Total 7

Question 2

Question Number	Scheme	Notes	Marks
8.(a)	$m=\frac{4k-8k}{k^2-4k^2}\left(=\frac{4}{3k}\right)$	Valid attempt to find gradient in terms of k	M1
	$y-8k=\frac{4}{3k}\left(x-4k^2\right)$ or $y-4k=\frac{4}{3k}\left(x-k^2\right)$ or $y=\frac{4}{3k}x+\frac{8k}{3}$	M1: Correct straight line method with their gradient in terms of k. If using $y=mx+c$ then award M provided they attempt to find c A1: Correct equation. If using $y=mx+c$, awardwhen they obtain $c=\frac{8k}{3}$ oe	M1A1
	$3ky-24k^2=4x-16k^2 \Rightarrow 3ky-4x=8k^2$ * or $3ky-12k^2=4x-4k^2 \Rightarrow 3ky-4x=8k^2$ *	Correct completion to printed answer with at least one intermediate step.	A1*
			(4)
(b)	(Focus) (4, 0)	Seen or implied as a number	B1
	(Directrix) $x = -4$	Seen or implied as a number	B1
	Gradient of l_2 is $-\frac{3k}{4}$	Attempt negative reciprocal of grad l_1 as a function of k	M1
	$y-0=\frac{-3k}{4}(x-4)$	Use of their changed gradient and numerical Focus in either formula, as printed oe	M1, A1
	$x=-4 \Rightarrow y=\frac{-3k}{4}(-4-4)$	Substitute numerical directrix into their line	M1
	$(y=)6k$	oe	A1
			(7)
			Total 11

Question 3

Question Number	Scheme	Marks
1(a)	$\dfrac{1}{3r-1}-\dfrac{1}{3r+2}$	M1 A1 (2)
(b)	$\displaystyle\sum_{r=1}^{n}\frac{3}{(3r-1)(3r+2)}=\frac{1}{2}-\frac{1}{5}+\frac{1}{5}-\frac{1}{8}+\frac{1}{8}-\frac{1}{11}+\ldots\frac{1}{3n-1}-\frac{1}{3n+2}$	M1 A1ft
	$=\dfrac{1}{2}-\dfrac{1}{3n+2}=\dfrac{3n}{2(3n+2)}$ *	A1 (3)
(c)	Sum = f(1000) – f(99) $\dfrac{3000}{6004}-\dfrac{297}{598}=0.00301$ or 3.01×10^{-3}	M1 A1 (2)
		7

Question 4

2.		
(a) (i)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x}{\sqrt{(1-x^2)}} + \arcsin x$	M1 A1 (2)
(ii)	At given value derivative $= \frac{1}{\sqrt{3}} + \frac{\pi}{6} = \frac{2\sqrt{3}+\pi}{6}$	B1 (1)
(b)	$\frac{dy}{dx} = \frac{6e^{2x}}{1+9e^{4x}}$	1M1 A1
	$= \frac{6}{e^{-2x}+9e^{2x}}$	2M1
	$= \frac{3}{\frac{5}{2}(e^{2x}+e^{-2x})+\frac{4}{2}(e^{2x}-e^{-2x})}$	3M1
	$\therefore \frac{dy}{dx} = \frac{3}{5\cosh 2x + 4\sinh 2x}$ *	A1 cso
		(5) **8**

Question 5

3. **(a)**	$x^2-10x+34=(x-5)^2+9$ so $\dfrac{1}{x^2-10x+34}=\dfrac{1}{(x-5)^2+9}=\dfrac{1}{u^2+9}$ (mark can be earned in either part (a) or (b))	B1
	$I=\int\dfrac{1}{u^2+9}du=\left[\dfrac{1}{3}\arctan\left(\dfrac{u}{3}\right)\right]$ | $I=\int\dfrac{1}{(x-5)^2+9}du=\left[\dfrac{1}{3}\arctan\left(\dfrac{x-5}{3}\right)\right]$	M1 A1
	Uses limits 3 and 0 to give $\dfrac{\pi}{12}$ | Uses limits 8 and 5 to give $\dfrac{\pi}{12}$	DM1 A1 (5)
(b) Alt 1	$I=\ln\left(\left(\dfrac{x-5}{3}\right)+\sqrt{\left(\dfrac{x-5}{3}\right)^2+1}\right)$ or $I=\ln\left(\dfrac{x-5+\sqrt{(x-5)^2+9}}{3}\right)$ or $I=\ln\left((x-5)+\sqrt{(x-5)^2+9}\right)$	M1 A1
	Uses limits 5 and 8 to give $\ln(1+\sqrt{2})$.	DM1 A1 (4) **9**
(b) Alt 2	Uses u = x-5 to get $I=\int\dfrac{1}{\sqrt{u^2+9}}du=\left[\text{ar}\sinh\left(\dfrac{u}{3}\right)\right]=\ln\left\{u+\sqrt{u^2+9}\right\}$	M1 A1
	Uses limits 3 and 0 and ln expression to give $\ln(1+\sqrt{2})$.	DM1 A1 (4)
(b) Alt 3	Use substitution $x-5=3\tan\theta$, $\dfrac{dx}{d\theta}=3\sec^2\theta$ and so $I=\int\sec\theta d\theta=\ln(\sec\theta+\tan\theta)$	M1 A1
	Uses limits 0 and $\dfrac{\pi}{4}$ to get $\ln(1+\sqrt{2})$.	DM1 A1 (4)

Question 6

Question Number	Scheme	Marks
7(a)	$\frac{dy}{dx} = \frac{dy}{dz}.\frac{dz}{dx}$ and $\frac{dy}{dz} = 2z$ so $\frac{dy}{dx} = 2z.\frac{dz}{dx}$	M1 M1 A1
	Substituting to get $2z.\frac{dz}{dx} - 4z^2 \tan x = 2z$ and thus $\frac{dz}{dx} - 2z \tan x = 1$ *	M1 A1 (5)
(b)	$\text{I.F.} = e^{\int -2\tan x dx} = e^{2\ln\cos x} = \cos^2 x$	M1 A1
	$\therefore \frac{d}{dx}\left(z\cos^2 x\right) = \cos^2 x \quad \therefore z\cos^2 x = \int \cos^2 x dx$	M1
	$\therefore z\cos^2 x = \int \frac{1}{2}(\cos 2x + 1)\,dx = \frac{1}{4}\sin 2x + \frac{1}{2}x + c$	M1 A1
	$\therefore z = \frac{1}{2}\tan x + \frac{1}{2}x\sec^2 x + c\sec^2 x$	A1 (6)
(c)	$\therefore y = (\frac{1}{2}\tan x + \frac{1}{2}x\sec^2 x + c\sec^2 x)^2$	B1ft (1)
		12

Question 7

Question Number	Scheme	Marks
3(a)	$(x+4)(x+3)^2-2(x+3)=0$, $(x+3)(x^2+7x+10)=0$ so $(x+2)(x+3)(x+5)=0$ or alternative method including calculator	M1
	Finds critical values –2 and -5	A1 A1
	Establishes $x > -2$	A1ft
	Finds and uses critical value –3 to give $-5 < x < -3$	M1A1 (6)