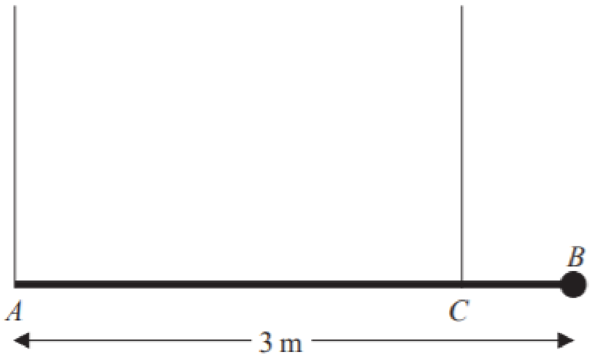
A-level Further Mechanics 1
Paper A

1. A plank AB has mass 40 kg and length 3 m. A load of mass 20 kg is attached to the plank at B. The loaded plank is held in equilibrium, with AB horizontal, by two vertical ropes attached at A and C, as shown in the figure below. The plank is modelled as a uniform rod and the load as a particle. Given that the tension in the rope at C is three times the tension in the rope at A, calculate,

a. The tension in the rope at C **[2]**

b. The distance CB **[5]**

(Total 7 marks)



2. Two forces **P** and **Q** act on a particle. The force **P** has magnitude 7 N and acts due north. The resultant of **P** and **Q** is a force of magnitude 10N acting in a direction with bearing 120°. Find,

i. The magnitude of **Q**

ii. The direction of **Q**, giving your answer as a bearing.

(Total 9 marks)

3. Three particles of mass $3m, 5m$ and γm are placed at points with coordinates (4, 0), (0, -3) and (4, 2) respectively. The centre of mass of the system of there particles is at $(2, k)$.

a. Show that $\gamma = 2$ **[4]**

b. Calculate the value of k **[3]**

(Total 7 marks)

4. A smooth sphere P of mass 2m is moving in a straight line with speed u on a smooth horizontal table. Another smooth sphere Q of mass m is at rest on the table. The sphere P collides directly with Q. The coefficient of restitution between P and Q is $\frac{1}{3}$. The spheres are modelled as particles.

a. Show that, immediately after the collision, the speeds of P and Q are $\frac{5}{9}u$ and $\frac{8}{9}u$ respectively. **[7]**

After the collision, Q strikes a fixed vertical wall which is perpendicular to the direction of motion of P and Q. The coefficient of restitution between Q and the wall is e. When P and Q collide again, P is brought to rest.

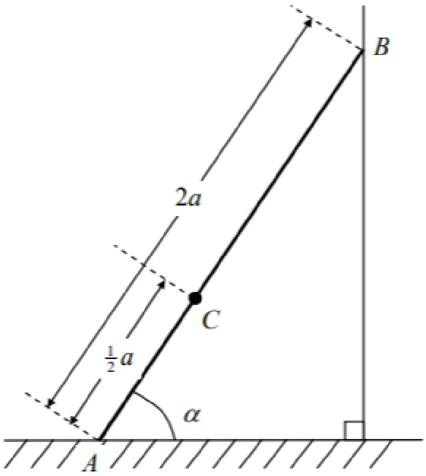
b. Find the value of e **[7]**

c. Explain why there must be a third collision between P and Q **[1]**

(Total 15 marks)

5. A uniform ladder AB, of mass m and length $2a$, has one end A on rough horizontal ground. The other end B rests against a smooth vertical wall. The ladder is in a vertical plane perpendicular to the wall. The ladder makes an angle α with the horizontal, where $tan\ \alpha\ = \frac{4}{3}$. A child of mass 2m stands on the ladder at C where $AC\ = \frac{1}{2}a$, as shown in the figure below. The ladder and the child are in equilibrium.

By modelling the ladder as a rod and the child as a particle, calculate the least possible value of the coefficient of friction between the ladder and the ground.



(Total 9 marks)

6. A car of mass 1000 kg is towing a trailer of mass 1500 kg along a straight horizontal road. The tow-bar joining the car to the trailer is modelled as a light rod parallel to the road. The total resistance to motion of the car is modelled as having constant magnitude 750 N. The total resistance to motion of the trailer is modelled as of magnitude R newtons, where R is a constant. When the engine of the car is working at a rate of 50 kW, the car and the trailer travel at a constant speed of 25 ms^{-1} .

a. Show that R = 1250 **[3]**

When travelling at 25 ms^{-1} the driver of the car disengages the engine and applies the brakes. The brakes provide a constant braking force of magnitude 1500 N to the car. The resisting forces of magnitude 750 N and 1250 N are assumed to remain unchanged. Calculate

b. The deceleration of the car while braking **[3]**

c. The thrust in the tow-bar while braking, **[2]**

d. The work done, in kJ, by the braking force in bringing the car and the trailer to rest. **[4]**

e. Suggest how the modelling assumption that the resistances to motion are constant could be refined to be more realistic. **[1]**

(Total 13 marks)

End of Paper

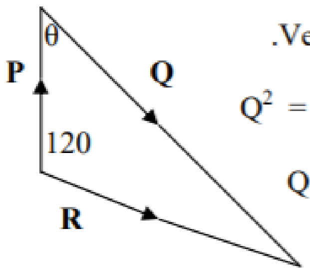
Total Marks: 60

<u>**Mark Scheme**</u>

Question 1

Question Number	Scheme	Marks
2		
	(a) R(↑): $T + 3T = 40g + 20g$	M1
	$T = 15g$, so tension at C is <u>$45g$ or 441 N or 440 N</u>	A1 (2)
	(b) M(B) $15g \times 3 + 45g \times d = 40g \times 1.5$	M1 A2,1,0√ ↓
	Solve: d = <u>1/3 or 0.33 or 0.333 m</u>	M1 A1 (5)

Question 2

4.	$\mathbf{R} = 10\sqrt{3}/2\,\mathbf{i} - 5\mathbf{j}$	M1 A1 ↓
	Using $\mathbf{P} = 7\mathbf{j}$ and $\mathbf{Q} = \mathbf{R} - \mathbf{P}$ to obtain $\mathbf{Q} = 5\sqrt{3}\mathbf{i} - 12\mathbf{j}$	M1 A1 ↓
	Magnitude = $\sqrt{[(5\sqrt{3})^2 + 12^2]} \approx$ <u>14.8 N</u> (AWRT)	↓ M1 A1
	angle with $\mathbf{i}$ = $\arctan(12/5\sqrt{3}) \approx 64.2°$	M1 A1
	bearing $\approx$ <u>144°</u> (AWRT)	A1 (9)
	Alternative method	
		
	.Vector triangle correct	B1
	$Q^2 = 10^2 + 7^2 + 2 \times 10 \times 7 \cos 60$	M1 A1
	Q $\approx$ <u>14.8 N</u> (AWRT)	A1
	$\dfrac{14.8}{\sin 120} = \dfrac{10}{\sin\theta}$	M1 A1 √ ↓
	$\Rightarrow \theta = 35.8, \Rightarrow$ bearing 144 (AWRT)	M1 A1, A1
		9

Question 3

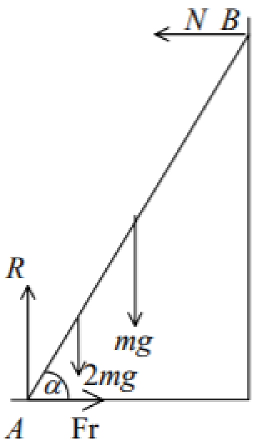
1.	(*a*)	Use of $(8 + \lambda)m$	B1
		i: $3m \times 4 + \lambda m \times 4 = (8 + \lambda)m \times 2$	M1
		Solving to $\lambda = 2$ (*)	M1 A1 (4)
		j: $5m \times (-3) + 2m \times 2 = 10m \times k$	M1 A1
		$k = -1.1$	A1 (3)
			(7 marks)

Question 4

6.	(*a*)	L.M. $2u = 2x + y$	M1 A1	
		NEL $y - x = \frac{1}{3}u$	M1 A1	
		Solving to $x = \frac{5}{9}u$ (*)	M1 A1	
		$y = \frac{8}{9}u$ (*)	A1	(7)
	(*b*)	$(\pm)\ \frac{8}{9}eu$	B1	
		L.M $\frac{10}{9}u - \frac{8}{9}eu = w$	M1 A1	
		NEL $w = \frac{1}{3}\left(\frac{5}{9}u + \frac{8}{9}eu\right)$	M1 A1	
		Solving to $e = \frac{25}{32}$ accept 0.7812s	M1 A1	(7)
	(*c*)	*Q* still has velocity and will *bounce back* from wall colliding with *stationary P*.	B1	(1)
				(15 marks)

Question 5

3.



(↑) $R = 3mg$	B1	
M(B) $mga\cos\alpha + 2mg \times \frac{3}{2}a\cos\alpha + \text{Fr}\times 2a\sin\alpha = R\times 2a\cos\alpha$	M1 A2 1,0	
Solving to $\text{Fr} = \frac{3}{4}mg$	M1 A1	
$\text{Fr} \le \mu R \Rightarrow \frac{3}{4}mg \le \mu\, 3mg$	M1	
$\mu \ge \frac{1}{4}$ (least value is $\frac{1}{4}$)	M1 A1	(9)

Question 6

5.	(a) $50000 = F \times 25 \ (F = 2000)$	or equivalent	M1		
	$\rightarrow \quad F = R + 750$		M1		
	$R = 1250$ *	cso	A1	3	
	(b) N2L $1500 + 2000 = 2500a$	ignore sign of a	M1 A1		
	$a = 1.4 \ (\text{m s}^{-2})$	cao	A1	3	
	(c) Trailer: $T + R = 1500 \times 1.4$ *or* Car: $T - 1500 - 750 = 1000 \times -1.4$		M1		
	$T = 850$ (N)		A1	2	
	(d) $25^2 = 2 \times 1.4 \times s \ (s = 223.2...)$		M1		
	$W = 1500 \times s$	ft their s	M1 A1ft		
	$= 335$ (kJ)	accept 330	A1	4	
	(e) Resistances vary with speeds		B1	1	**13**