

Solutions

1a.

$AE = \sqrt{(15^2 + 20^2 + 0^2)} = 25$	M1A1
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1b.

$\overline{AE} = \begin{pmatrix} 15 \\ -20 \\ 0 \end{pmatrix} = 5 \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix}$ Any correct form	M1
Equation of BD is $\mathbf{r} = \begin{pmatrix} -1 \\ -7 \\ 11 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} -1 \\ -7 \\ 11 \end{pmatrix} + \lambda \begin{pmatrix} 15 \\ -20 \\ 0 \end{pmatrix}$	A1
BD = 15 $\Rightarrow \lambda = 3$ $\lambda = 3$ or 3/5 as appropriate	M1
$\Rightarrow D$ is (8, -19, 11) cao	A1

1c.

One verification	M1
At A: $-3 \times 0 + 4 \times 0 + 5 \times 6 = 30$ (OR B1 Normal, M1 scalar product with 1 vector in At B: $-3 \times (-1) + 4 \times (-7) + 5 \times 11 = 30$ the plane, A1 two correct, A1 verification with a At C: $-3 \times (-8) + 4 \times (-6) + 5 \times 6 = 30$ point	A2,1,0
Normal is $\begin{pmatrix} -3 \\ 4 \\ 5 \end{pmatrix}$ OR M1 vector form of equation of plane eg $\mathbf{r} = 0\mathbf{i} + 0\mathbf{j} + 6\mathbf{k} + \mu(\mathbf{i} + 7\mathbf{j} - 5\mathbf{k}) + \nu(8\mathbf{i} + 6\mathbf{j} + 0\mathbf{k})$ M1 elimination of both parameters A1 equation of plane B1 Normal *)	B1



Solutions

1.

$\overrightarrow{AB} = (2\mathbf{i} - \mathbf{j} + 2\mathbf{k}), \overrightarrow{CB} = (-2\mathbf{i} + 2\mathbf{j} + \mathbf{k}),$ (or $\overrightarrow{BA}, \overrightarrow{BC},$ or $\overrightarrow{AB}, \overrightarrow{BC}$ stated in above form or column vector form.	M1A1
$\cos \hat{ABC} = \frac{CB \bullet AB}{ CB AB } = -\frac{4}{9}$ or (-0.444)	M1A1

2.

$\overrightarrow{BA} = \begin{pmatrix} -4 \\ 1 \\ -3 \end{pmatrix}, \overrightarrow{BC} = \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix}$ soi, condone wrong sense	B1
$\overrightarrow{BA} \cdot \overrightarrow{BC} = \begin{pmatrix} -4 \\ 1 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix} = (-4) \times 2 + 1 \times 5 + (-3) \times (-1)$ <div style="text-align: right; margin-right: 50px;">scalar product</div>	M1
$= -8 + 5 + 3 = 0$ \Rightarrow angle $ABC = 90^\circ$ (= 0)	A1
Area of triangle = $\frac{1}{2} \times BA \times BC$ area of triangle formula oe	M1
$= \frac{1}{2} \times \sqrt{(-4)^2 + 1^2 + 3^2} \times \sqrt{2^2 + 5^2 + (-1)^2}$	M1
$= \frac{1}{2} \times \sqrt{26} \times \sqrt{30}$ length formula	M1
$= 13.96$ sq units accept 14.0 and $\sqrt{195}$	A1



Solutions

1a.

$\mathbf{r} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} + \dots$ need \mathbf{r} (or another letter) = or $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ for first B1	B1
$\dots + \lambda \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$ NB answer is not unique eg $\mathbf{r} = \begin{pmatrix} -2 \\ 2 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$ Accept i/j/k form and condone row vectors.	B1

1b.

$x + 3y + 2z = 4$ $\Rightarrow -2\lambda + 3(1+\lambda) + 2(3+2\lambda) = 4$ clear)	substituting their line in plane equation (condone a slip if intention)	M1
$\Rightarrow 5\lambda = -5, \lambda = -1$ www cao	NB λ is not unique as depends on choice of line in (i)	A1
so point of intersection is (2, 0, 1) www cao		A1

1c.

Angle between $-2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ is θ where ...	M1
$\cos\theta = \frac{-2 \times 1 + 1 \times 3 + 2 \times 2}{\sqrt{9} \sqrt{14}} = \frac{5}{3\sqrt{14}}$ correct formula (including cosine) , with substitution, for these vectors	M1
$\Rightarrow \theta = 63.5^\circ$ www cao (63.5 in degrees (or better) or 1.109 radians or better)	A1



Solutions

1. Either

For correct parametric form for either line	B1
For 3 equations using 2 different parameters	M1
$\begin{cases} 1 + s = -3 + 2t \\ -2 - s = 1 + 3t \\ -4 + 2s = 5 + 4t \end{cases} \Rightarrow \begin{cases} s - 2t = -4 \\ s + 3t = -3 \\ 2s - 4t = 9 \end{cases}$	A1
For attempting to solve to show (in)consistency	M1
\Rightarrow contradiction, so skew For correct conclusion	A1

Or

line segment between l_1 and $l_2 = \pm[4, -3, -9]$ For correct vector	B1
$\mathbf{n} = [1, -1, 2] \times [2, 3, 4] = (\pm)[-2, 0, 1]$ For finding vector product of direction vectors	M1
$\text{distance} = \frac{ [4, -3, -9] \cdot [-2, 0, 1] }{\left(\sqrt{2^2 + 0^2 + 1^2}\right)} = \frac{17}{\sqrt{5}}$	A1
For using numerator of distance formula	M1
$\neq 0$, so skew For correct scalar product and correct conclusion	A1

2. Either

$(\mathbf{r} =)[3 + t, 1 + 4t, -2 + 2t]$ For parametric form of l seen or implied	M1
$8(3 + t) - 7(1 + 4t) + 10(-2 + 2t) = 7$ For substituting into plane equation	M1A1
$\Rightarrow (0t) + (-3) = 7 \Rightarrow$ contradiction For obtaining a contradiction	A1
l is parallel to Π , no intersection For conclusion from correct working	B1

Or

$[1, 4, 2] \cdot [8, -7, 10] = 0$ For finding scalar product of direction vectors	M1
$\Rightarrow l$ is parallel to Π For correct conclusion	A1
$(3, 1, -2)$ into Π For substituting point into plane equation	M1
$\Rightarrow 24 - 7 - 20 \neq 7$ For obtaining a contradiction	A1
l is parallel to Π , no intersection For conclusion from correct working	B1

Or

Solve $\frac{x-3}{1} = \frac{y-1}{4} = \frac{z+2}{2}$ and $8x - 7y + 10z = 7$ For eliminating one variable	M1A1
For eliminating another variable	M1
eg $4z + 4 = 4z + 8$ For obtaining a contradiction	A1
l is parallel to Π , no intersection For conclusion from correct working	B1



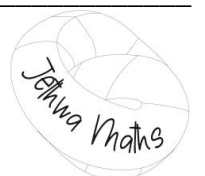


1. Two skew lines have equations $\frac{x}{2} = \frac{y+3}{1} = \frac{z-6}{3}$ and $\frac{x-5}{3} = \frac{y+1}{1} = \frac{z-7}{5}$.

a. Find the direction of the common perpendicular to the lines. (2)

b. Find the shortest distance between the lines. (4)

2. The plane Π_1 has vector equation $\mathbf{r} \cdot (3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) = 5$. Find the perpendicular distance from the point $(6, 2, 12)$ to the plane Π_1 . (3)



Solutions

1. a

$\mathbf{n} = [2, 1, 3] \times [3, 1, 5]$ $= [2, -1, -1]$	For using direction vectors and attempt to find vector product For correct direction (allow multiples)	M1
		A1

b

$d = \frac{ [5, 2, 1] \cdot [2, -1, -1] }{\sqrt{6}}$	For $(\mathbf{AB} \Rightarrow) [5, 2, 1]$ or any vector joining lines	B1
For attempt at evaluating $\mathbf{AB} \cdot \mathbf{n}$		M1
For $ \mathbf{n} $ in denominator		M1
$= \frac{7}{\sqrt{6}} = \frac{7}{6}\sqrt{6} = 2.8577$		A1

2. Either

$(6\mathbf{i} + 2\mathbf{j} + 12\mathbf{k}) \cdot (3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) = 34$	Attempt scalar product	M1
$\frac{ (6\mathbf{i} + 2\mathbf{j} + 12\mathbf{k}) \cdot (3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) - 5 }{\sqrt{3^2 + 4^2 + 2^2}}$	Use of correct formula	M1
$\sqrt{29}$ (not $-\sqrt{29}$)	Correct distance (Allow $29/\sqrt{29}$)	A1

Or

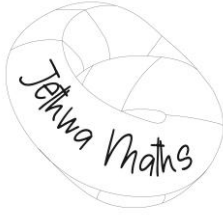
$\mathbf{r} = (6\mathbf{i} + 2\mathbf{j} + 12\mathbf{k}) + \lambda(3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})$ $\therefore 6 + 3\lambda \quad 3 + 2 - 4\lambda \quad -4 + 12 + 2\lambda \quad 2 = 5$ Substitutes the parametric coordinates of the line through $(6, 2, 12)$ perpendicular to the plane into the cartesian equation.	Solves for λ to obtain the required point or vector.	M1
$\lambda = -1 \Rightarrow 3, 6, 10$ or $-3\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$	Correct distance	M1
$\sqrt{29}$		A1

Or

Parallel plane containing $(6, 2, 12)$ is $\mathbf{r} \cdot (3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) = 34$ $\Rightarrow \frac{\mathbf{r} \cdot (3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})}{\sqrt{29}} = \frac{34}{\sqrt{29}}$	Origin to this plane is $\frac{34}{\sqrt{29}}$	M1
$\Rightarrow \frac{\mathbf{r} \cdot (3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})}{\sqrt{29}} = \frac{5}{\sqrt{29}}$	Origin to plane is $\frac{5}{\sqrt{29}}$	M1
$\frac{34}{\sqrt{29}} - \frac{5}{\sqrt{29}} = \sqrt{29}$		A1



Further Maths
A-Level Starter
Activity



Topic: Finding Perpendiculars (5)

Chapter Reference: Core Pure 1, Chapter 9

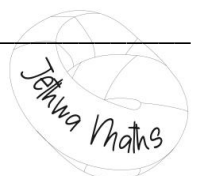
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minutes**

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Solutions

1a.

$\mathbf{n} = [2, 1, 3] \times [3, 1, 5]$ For using direction vectors and attempt to find vector product $= [2, -1, -1]$ For correct direction (allow multiples)	M1
	A1

1b.

$d = \frac{ [5, 2, 1] \cdot [2, -1, -1] }{\sqrt{6}}$ For $(\mathbf{AB} \Rightarrow) [5, 2, 1]$ or any vector joining lines	B1
For attempt at evaluating $\mathbf{AB} \cdot \mathbf{n}$	M1
For $ \mathbf{n} $ in denominator	M1
$= \frac{7}{\sqrt{6}} = \frac{7}{6}\sqrt{6} = 2.8577$ For correct distance	A1

2. Either

$(6\mathbf{i} + 2\mathbf{j} + 12\mathbf{k}) \cdot (3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) = 34$ Attempt scalar product	M1
$\frac{ (6\mathbf{i} + 2\mathbf{j} + 12\mathbf{k}) \cdot (3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) - 5 }{\sqrt{3^2 + 4^2 + 2^2}}$ Use of correct formula	M1
$\sqrt{29}$ (not $-\sqrt{29}$) Correct distance (Allow $29/\sqrt{29}$)	A1

Or

$\mathbf{r} = (6\mathbf{i} + 2\mathbf{j} + 12\mathbf{k}) + \lambda(3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})$ $\therefore 6 + 3\lambda \quad 3 + 2 - 4\lambda \quad -4 + 12 + 2\lambda \quad 2 = 5$ Substitutes the parametric coordinates of the line through $(6, 2, 12)$ perpendicular to the plane into the cartesian equation.	M1
$\lambda = -1 \Rightarrow 3, 6, 10$ or $-3\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$ Solves for λ to obtain the required point or vector.	M1
$\sqrt{29}$ Correct distance	A1

Or

Parallel plane containing $(6, 2, 12)$ is $\mathbf{r} \cdot (3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) = 34$ $\Rightarrow \frac{\mathbf{r} \cdot (3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})}{\sqrt{29}} = \frac{34}{\sqrt{29}}$ Origin to this plane is $\frac{34}{\sqrt{29}}$	M1
$\Rightarrow \frac{\mathbf{r} \cdot (3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})}{\sqrt{29}} = \frac{5}{\sqrt{29}}$ Origin to plane is $\frac{5}{\sqrt{29}}$	M1
$\frac{34}{\sqrt{29}} - \frac{5}{\sqrt{29}} = \sqrt{29}$ Correct distance	A1

