A-Level Starter Activity

Jethwa Maths

Topic: Algebraic Fractions

Chapter Reference: Pure 1, Chapter 7

7 minutes

1. Show that $\frac{1}{2x^2+x-15} \div \frac{1}{3x^2+9x}$ simplifies to $\frac{ax}{bx+c}$, where a, b and c are integers to be found. **(2)**

2. Solve $\frac{6}{x-1} - \frac{6}{x+1} = 1$ **(4)**

3. Simplify fully, $\frac{5}{2x-6} - \frac{x+2}{x^2-4x+3}$ **(4)**

Solutions

1.

$\frac{1}{2x^2+x-15} \div \frac{1}{3x^2+9x}$ $= \frac{1}{(2x-5)(x+3)} \times \frac{3x(x+3)}{1}$	**M1**
$= \frac{3x}{2x-5}$	**M1**

2.

$\frac{6}{x-1} - \frac{6}{x+1} = 1$ $\frac{6(x+1)-6(x-1)}{(x-1)(x+1)} = 1$	**M1**
$\frac{6x+6-6x+6}{x^2+1} = 1$ $\frac{12}{x^2+1} = 1$	**M1**
$12 = x^2 + 1$ $x^2 = 11$	**M1**
$x = \pm\sqrt{11}$	**M1**

3.

$\frac{5}{2x-6} - \frac{x+2}{x^2-4x+3} = \frac{5}{2(x-3)} - \frac{x+2}{(x-3)(x-1)}$	**M1**
$= \frac{5(x-1)-2(x+2)}{2(x-3)(x-1)}$ $= \frac{5x-5-2x-4}{2(x-3)(x-1)}$ $= \frac{3x-9}{2(x-3)(x-1)}$	**M1**
$= \frac{3(x-3)}{2(x-3)(x-1)}$	**M1**
$= \frac{3}{2(x-1)}$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Dividing Polynomials Chapter Reference: Pure 1, Chapter 7	6 minutes

1. Find the quotient obtained by dividing $(x^2 + 2x^2 - x - 2)$ by $(x + 1)$ **(3)**

2. Find the quotient by dividing $(20 + x + 3x^2 + x^3)$ by $(x + 4)$ **(4)**

Solutions

1.

Attempt to divide **M1**
x^2 term, **M1**
Full quotient **M1**

$$
\begin{array}{r|rrrr}
 & x^2 & +x & -2 & \\
\hline
x+1 & x^3 & +2x^2 & -x & -2 \\
 & x^3 & +x^2 & & \\
\cline{2-3}
 & & x^2 & -x & \\
 & & x^2 & +x & \\
\cline{3-4}
 & & & -2x & -2 \\
 & & & -2x & -2 \\
\cline{4-5}
\end{array}
$$

quotient: $x^2 + x - 2$

2.

Rewriting $20 + x + 3x^2 + x^3$ as $x^3 + 3x^2 + x + 20$ **M1**
Attempt to divide **M1**
x^2 term, **M1**
Full quotient **M1**

$$
\begin{array}{r|rrrr}
 & x^2 & -x & +5 & \\
\hline
x+4 & x^3 & +3x^2 & +x & +20 \\
 & x^3 & +4x^2 & & \\
\cline{2-3}
 & & -x^2 & +x & \\
 & & -x^2 & -4x & \\
\cline{3-4}
 & & & 5x & +20 \\
 & & & 5x & +20 \\
\cline{4-5}
\end{array}
$$

quotient: $x^2 - x + 5$

A-Level Starter Activity

Jethwa Maths

Topic: Proof by Counter Example Chapter Reference: Pure 1, Chapter 7	6 minutes

1. 'If m and n are irrational numbers, where $m \neq n$, then mn is also irrational.'
Disprove this statement by means of a counter example. **(2)**

2. 'For every real number x, $(x + 1)^2 = x^2 + 1$.'
Disprove this statement by means of a counter example.

3. 'If n is an integer and n^2 is divisible by 4, then n is divisible by 4'.
Disprove this statement by means of a counter example.

Solutions

1.

$3\sqrt{2} = \sqrt{(9)(2)} = \sqrt{18}$	**M1**
$\sqrt{2}\,\sqrt{18} = \sqrt{36} = 6$ So if *m* and *n* are irrational numbers then *mn* is not always irrational.	**M1**

2.

$(x + 1)^2 = x^2 + 1$ When $x = -2$	**M1**
$(-2 + 1)^2 = (-2)^2 + 1$ $1 \neq 5$ Therefore, proof by counter example.	**M1**

3.

Consider $n = 6$ (or suitable alternative)	**M1**
$n^2 = 36$ n^2 is divisible by 4, but n is not. Therefore, proof by counter example.	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: The Factor Theorem

Chapter Reference: Pure 1, Chapter 7

7 minutes

1. Determine whether $(x - 1)$ is a factor of $(x^3 + 2x^2 - 2x - 1)$ **(2)**

2. Determine whether $(3x - 2)$ is a factor of $(2 - 17x + 25x^2 - 6x^3)$ **(2)**

3. $g(x) = x^3 + 7x^2 + 7x - 6$.
Given that $x = -2$, is a solution to the equation $g(x) = 0$,
a. Express $g(x)$, as the product of a linear factor and a quadratic factor. **(4)**
b. Find, to 2 decimal places, the other two solutions to the equation, $g(x) = 0$. **(1)**

Solutions

1.

Let $f(x) = x^3 + 2x^2 - 2x - 1$ Find f(1)	**M1**
$f(1) = 1 + 2 - 2 - 1 = 0$ Therefore, $(x - 1)$ is a factor.	**M1**

2.

Let $f(x) = 2 - 17x + 25x^2 - 6x^3$ Find $f(\frac{2}{3})$	**M1**
$f(\frac{2}{3}) = 2 - \frac{34}{3} + \frac{100}{9} - \frac{16}{9} = 0$ Therefore $(3x - 2)$ is a factor	**M1**

3a.

$g(-2) = 0$, Therefore $(x + 2)$ is a factor of $g(x)$	**M1**
$x^2 + 5x - 3$ $x + 2 \overline{)\, x^3 + 7x^2 + 7x - 6}$ $x^3 + 2x^2$ $5x^2 + 7x$ $5x^2 + 10x$ $-3x - 6$ $-3x - 6$ Attempt to divide **M1** Obtaining $x^2 + 5x - 3$ **M1**	
Therefore, $g(x) = (x + 2)(x^2 + 5x - 3)$	**M1**

3b.

Other solutions given by, $x^2 + 5x - 3 = 0$ $x = -5.54, 0.54$	**M1**