

Solutions

1a.

$G_Y(1) = 1$ $k(1 + 2 + 3)^2 = 1$ $36k = 1$	M1
$k = \frac{1}{36}$	(oe) A1

1b.

$G_Y(t) = \frac{1}{36}(1 + 2t + 3t^2)^2$ $G_Y(t) = \frac{1}{36}(1 + 4t + 10t^2 + 12t^3 + 9t^4)$	M1
$P(Y = 2) = \frac{1}{36} \times 10 = \frac{10}{36} = \frac{5}{18}$	A1

2a.

$G_X(1) = 1$ $k(1 + 1 + 3)^2 = 1$ $25k = 1$	M1
$k = \frac{1}{25}$	(oe) A1

2b.

$G_X(t) = \frac{1}{25}(1 + t + 3t^2)^2$ $G_X(t) = \frac{1}{25}(1 + 2t + 7t^2 + 6t^3 + 9t^4)$	M1												
$G_X(t) = \frac{1}{25} + \frac{2}{25}t + \frac{7}{25}t^2 + \frac{6}{25}t^3 + \frac{9}{25}t^4$	M1												
<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="padding: 5px;">x</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">2</td> <td style="padding: 5px;">3</td> <td style="padding: 5px;">4</td> </tr> <tr> <td style="padding: 5px;">P(X = x)</td> <td style="padding: 5px;">$\frac{1}{25}$</td> <td style="padding: 5px;">$\frac{2}{25}$</td> <td style="padding: 5px;">$\frac{7}{25}$</td> <td style="padding: 5px;">$\frac{6}{25}$</td> <td style="padding: 5px;">$\frac{9}{25}$</td> </tr> </table>	x	0	1	2	3	4	P(X = x)	$\frac{1}{25}$	$\frac{2}{25}$	$\frac{7}{25}$	$\frac{6}{25}$	$\frac{9}{25}$	A1
x	0	1	2	3	4								
P(X = x)	$\frac{1}{25}$	$\frac{2}{25}$	$\frac{7}{25}$	$\frac{6}{25}$	$\frac{9}{25}$								



Solutions

1a.

$\begin{aligned}G_X(t) &= (1 - p + pt)^n \\ &= (1 - 0.2 + 0.2t)^5 \\ &= (0.8 + 0.2t)^5 \\ &= 0.2^5(4 + t)^5\end{aligned}$	M1
$= \frac{1}{3125}(4 + t)^5$	A1

1b.

$\begin{aligned}G_X(t) &= e^{\lambda(t-1)} \\ &= e^{1.5(t-1)}\end{aligned}$	A1
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1c.

$\begin{aligned}G_X(t) &= \frac{pt}{1-(1-p)t} \\ &= \frac{0.9t}{1-(1-0.9)t} \\ &= \frac{0.9t}{1-0.1t} \\ &= \frac{9t}{10-t}\end{aligned}$	M1
	A1

1d.

$\begin{aligned}G_X(t) &= \left(\frac{pt}{1-(1-p)t}\right)^r \\ &= \left(\frac{0.2t}{1-(1-0.2)t}\right)^6 \\ &= \left(\frac{0.2t}{1-0.8t}\right)^6 \\ &= \left(\frac{2t}{10-8t}\right)^6 = \left(\frac{t}{5-4t}\right)^6\end{aligned}$	M1
	A1

2a.

$X \sim \text{Po}(0.7)$	A1
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2b.

$\begin{aligned}G_X(t) &= e^{\lambda(t-1)} \\ &= e^{0.7(t-1)}\end{aligned}$	A1
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Solutions

1a.

$G_Y(t) = \frac{10}{(7-t)^2} = 10(7-t)^{-2}$	M1
$G'_Y(t) = -1 \times -2 \times 10(7-t)^{-3} = \frac{20}{(7-t)^3}$	
$E(Y) = G'_Y(1) = \frac{5}{54}$	A1
$G''_Y(t) = -1 \times -3 \times 20(7-t)^{-4} = \frac{60}{(7-t)^4}$	M1
$G''_Y(1) = \frac{5}{108}$	M1
Hence $\text{Var}(Y) = G''_Y(1) + G'_Y(1) + (G'_Y(1))^2$ $= \frac{5}{108} + \frac{5}{54} - \frac{25}{2916}$ $= \frac{95}{729}$	A1

1b.

Standard deviation of $Y = \sqrt{\text{Var}(Y)} = \sqrt{\frac{95}{729}} = \frac{\sqrt{95}}{27} = 0.361$ (3.d.p)	(oe)	A1
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Solutions

1a.

$G'_Y(t) = 0.18t + 0.72t^2 + 1.36t^3 + 1.2t^4 + 0.54t^5$ ≥ 3 terms correct	M1
Sub $t=1$, $(0.18 + 0.72 + 1.36 + 1.2 + 0.54)$	M1
$= 4$ (E(Y))	A1
$G''_Y(t) = 0.18 + 1.44t + 4.08t^2 + 4.8t^3 + 2.7t^4$	M1
Sub $t=1$, $0.18 + 1.44 + 4.08 + 4.8 + 2.7 = 13.2$	M1
Use correct formula for Var, $13.2 + 4 - 16$	
$= 1.2$	A1

1b.

Attempt to factorise into $(0.3t + \dots)(0.3t + \dots)$	M1M1
Attempt to find $\sqrt{(G_Y(t))}$ seen or implied.	
$= 0.3t + 0.4t^2 + 0.3t^3$	A1

1c.

0.4	A1
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Solutions

1.

$G_Z(t) = G_X(t)G_Y(t)$ $= \frac{1}{2}(t + t^2) \times \frac{2}{3}(1 + 2t + 3t^2)^2$	M1
$= \frac{1}{3}(t + t^2)(1 + 2t + 3t^2)^2$	A1

2a.

$G'_X(t) = t + 4t^3$	M1
$G_X(1) = 1 + 4$	M1
$= 5$	A1

2b.

$G_Y(t) = t^{-1}(G_X(t^2))$ $= t^{-1}\left(\frac{1}{4} + \frac{1}{2}t^4 + t^8\right)$	M1
$= \frac{1}{4}t^{-1} + \frac{1}{2}t^3 + t^7$	A1

