Topic: Methods in Differential Equations (1)
Chapter Reference: Core Pure 2, Chapter 7

8 minutes

1. Find the general solution of the differential equation

$$\sin x\frac{dy}{dx} - y\cos x = \sin 2x \sin x,$$

giving your answer in the form $y = f(x)$. **(8)**

Solutions

1.

$\sin x \frac{dy}{dx} - y\cos x = \sin 2x \sin x$	**M1**
$\frac{dy}{dx} - \frac{y\cos x}{\sin x} = \frac{\sin 2x \sin x}{\sin x}$	
$\frac{dy}{dx} - \frac{y\cos x}{\sin x} = \sin 2x$	
Integrating factor $= e^{\int -\frac{\cos x}{\sin x}} = e^{-\ln \sin x}$	**M1** **A1** **A1**
$= \frac{1}{\sin x}$	
$\left(\frac{1}{\sin x}\right)\frac{dy}{dx} - \frac{y\cos x}{\sin^2 x} = \frac{\sin 2x}{\sin x}$	
$\frac{d}{dx}\left(\frac{y}{\sin x}\right) = \sin 2x \times \frac{1}{\sin x}$	**M1**
$\frac{d}{dx}\left(\frac{y}{\sin x}\right) = 2\cos x$	**A1**
$\frac{y}{\sin x} = \int 2\cos x \, dx$	
$y = 2\sin^2 x + K\sin x$	**A1**

Topic: Methods in Differential Equations (2)
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8 minutes

1. Find the general solution of the differential equation $\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + 4x = 0$, where k is a real constant, in each of the following cases,

a. $|k| > 2$ **(4)**

b. $|k| < 2$ **(2)**

c. $|k| = 2$ **(2)**

Solutions

1a.

$m^2 + 2km + 4 = 0$ $\Rightarrow m - k \pm \sqrt{k^2 - 4}$ $x = e^{-kt}\left(Ae^{\sqrt{k^2-4}\,t} + Be^{-\sqrt{k^2-4}\,t}\right)$	**M1** **A1** **M1** **A1**

1b.

$x = e^{-kt}\left(Ae^{i\sqrt{4-k^2}\,t} + Be^{-i\sqrt{4-k^2}\,t}\right)$ $x = e^{-kt}\left(A'\cos\sqrt{4-k^2}\,t + B'\sin\sqrt{4-k^2}\,t\right)$	**M1** **A1**

1c.

$x = e^{-2t}(A'' + B''t)$	**M1** **A1**

Further Maths A-Level Starter Activity

Topic: Methods in Differential Equations (3)
Chapter Reference: Core Pure 2, Chapter 7

8 minutes

1a. Find the value of λ for which $\lambda t^2 e^{3t}$ is the particular integral of the differential equation

$$\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 9y = 6e^{3t}, \qquad t \geq 0$$

(5)

1b. Hence find the general solution of this differential equation. **(3)**

Solutions

1a.

$y = \lambda t^2 e^{3t}$ $\frac{dy}{dt} = 2\lambda t e^{3t} + 3\lambda t^2 e^{3t}$ $\frac{d^2y}{dt^2} = 2\lambda e^{3t} + 6\lambda t e^{3t} + 6\lambda t e^{3t} + 9\lambda t^2 e^{3t}$ $2\lambda e^{3t} + 6\lambda t e^{3t} + 6\lambda t e^{3t} + 9\lambda t^2 e^{3t} - 12\lambda t e^{3t} - 18\lambda t^2 e^{3t} + 9\lambda t^2 e^{3t} = 6e^{3t}$ $\lambda = 3$	**M1A1** **A1** **M1** **A1**

1b.

$m^2 - 6m + 9 = 0$ $(m-3)^2 = 0$ C.F. $y = (A + 3t)e^{3t}$ G.S. $y = (A + 3t)e^{3t} + 3t^2 e^{3t}$	**M1A1** **A1**

1. The differential equation

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = \cos 3t, \qquad t \geq 0$$

describes the motion of a particle along the x- axis.

Find the general solution of this differential equation. **(8)**

Solutions

1.

$m^2 + 6m + 9 = 0 \qquad m = -3$	**M1**
C.F. $x = (A + Bt)e^{-3t}$	**A1**
P.I. $x = P\cos 3t + Q\sin 3t$	**B1**
$\dot{x} = -3P\cos 3t + 3Q\sin 3t$	**M1**
$\ddot{x} = -9P\cos 3t - 9Q\sin 3t$	
$(-9P\cos 3t - 9Q\sin 3t) + 6(-3P\cos 3t + 3Q\sin 3t) + 9(P\cos 3t + Q\sin 3t) = \cos 3t$	**M1**
$-9P + 18Q + 9P = 1$ and $-9Q - 18P + 9Q = 0$	**M1**
$p = 0 \qquad and \qquad Q = \frac{1}{18}$	**A1**
$x = (A + Bt)e^{-3t} + \frac{1}{18}\sin 3t$	**A1**

Topic: Methods in Differential Equations (5)

Chapter Reference: Core Pure 2, Chapter 7

8 minutes

1.

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 2e^{-t}$$

Given that $x = 0$ and $\frac{dx}{dt} = 2$ at $t = 0$,

Find x in terms of t. **(8)**

Solutions

1.

$m^2 + 5m + 6$ $\quad\quad\quad\quad$ $m = -3, -2$	**M1** **A1**
C.F. $x = Ae^{-3t} + Be^{-2t}$	
$x = ke^{-t} \Rightarrow \dfrac{dx}{dt} = -ke^{-t} \Rightarrow \dfrac{d^2x}{dt^2} = ke^{-t}$	**M1**
$\Rightarrow ke^{-t} + 5(-ke^{-t}) + 6(ke^{-t}) = 2e^{-t}$ $k = 1$	**A1**
P.I. $x = e^{-t}$	
$x = Ae^{-3t} + Be^{-2t} + e^{-t}$	**M1**
$\dfrac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t} - e^{-t}$	**M1**
$0 = A + B + 1$ $2 = -3A - 2B - 1$	**M1**
$A = -1, B = 0$	
So, $x = e^{-3t} + e^{-t}$	**A1**