

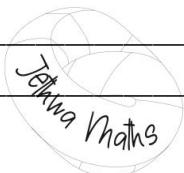


1. Find the general solution of the differential equation

$$\sin x \frac{dy}{dx} - y \cos x = \sin 2x \sin x,$$

giving your answer in the form $y = f(x)$.

(8)



Solutions

1.

$$\sin x \frac{dy}{dx} - y \cos x = \sin 2x \sin x$$

M1

$$\frac{dy}{dx} - \frac{y \cos x}{\sin x} = \frac{\sin 2x \sin x}{\sin x}$$

$$\frac{dy}{dx} - \frac{y \cos x}{\sin x} = \sin 2x$$

$$\text{Integrating factor} = e^{\int -\frac{\cos x}{\sin x} dx} = e^{-\ln \sin x}$$

M1**A1****A1**

$$= \frac{1}{\sin x}$$

$$\left(\frac{1}{\sin x}\right) \frac{dy}{dx} - \frac{y \cos x}{\sin^2 x} = \frac{\sin 2x}{\sin x}$$

$$\frac{d}{dx} \left(\frac{y}{\sin x} \right) = \sin 2x \times \frac{1}{\sin x}$$

M1

$$\frac{d}{dx} \left(\frac{y}{\sin x} \right) = 2 \cos x$$

A1

$$\frac{y}{\sin x} = \int 2 \cos x \, dx$$

A1

$$y = 2 \sin^2 x + K \sin x$$

Further Maths A-Level Starter Activity



Topic: Methods in Differential Equations (2)

Chapter Reference: Core Pure 2, Chapter 7

8

minutes

1. Find the general solution of the differential equation $\frac{d^2x}{dt^2} + 2k \frac{dx}{dt} + 4x = 0$, where k is a real constant, in each of the following cases,

a. $|k| > 2$

(4)

b. $|k| < 2$

(2)

c. $|k| = 2$

(2)



Solutions

1a.

$$m^2 + 2km + 4 = 0$$
$$\Rightarrow m - k \pm \sqrt{k^2 - 4}$$

$$x = e^{-kt} (Ae^{\sqrt{k^2-4}t} + Be^{-\sqrt{k^2-4}t})$$

M1
A1
M1
A1

1b.

$$x = e^{-kt} (Ae^{i\sqrt{4-k^2}t} + Be^{-i\sqrt{4-k^2}t})$$

$$x = e^{-kt} (A' \cos \sqrt{4-k^2}t + B' \sin \sqrt{4-k^2}t)$$

M1
A1

1c.

$$x = e^{-2t} (A'' + B''t)$$

M1
A1



Further Maths A-Level Starter Activity



Topic: Methods in Differential Equations (3)

Chapter Reference: Core Pure 2, Chapter 7

8

minutes

- 1a. Find the value of λ for which $\lambda t^2 e^{3t}$ is the particular integral of the differential equation

$$\frac{d^2y}{dt^2} - 6 \frac{dy}{dt} + 9y = 6e^{3t}, \quad t \geq 0$$

(5)

- 1b. Hence find the general solution of this differential equation.

(3)



Solutions

1a.

$$y = \lambda t^2 e^{3t}$$

$$\frac{dy}{dt} = 2\lambda t e^{3t} + 3\lambda t^2 e^{3t}$$

$$\frac{d^2y}{dt^2} = 2\lambda e^{3t} + 6\lambda t e^{3t} + 6\lambda t^2 e^{3t} + 9\lambda t^2 e^{3t}$$

$$2\lambda e^{3t} + 6\lambda t e^{3t} + 6\lambda t^2 e^{3t} + 9\lambda t^2 e^{3t} - 12\lambda t e^{3t} - 18\lambda t^2 e^{3t} + 9\lambda t^2 e^{3t} = 6e^{3t}$$

$$\lambda = 3$$

M1A1

A1

M1

A1

1b.

$$m^2 - 6m + 9 = 0$$

$$(m - 3)^2 = 0$$

$$\text{C.F. } y = (A + 3t)e^{3t}$$

$$\text{G.S. } y = (A + 3t)e^{3t} + 3t^2 e^{3t}$$

M1A1

A1



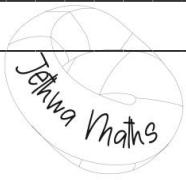
1. The differential equation

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = \cos 3t, \quad t \geq 0$$

describes the motion of a particle along the x - axis.

Find the general solution of this differential equation.

(8)



Solutions

1.

$$m^2 + 6m + 9 = 0 \quad m = -3$$

$$\text{C.F. } x = (A + Bt)e^{-3t}$$

$$\text{P.I. } x = P \cos 3t + Q \sin 3t$$

$$\dot{x} = -3P \cos 3t + 3Q \sin 3t$$

$$\ddot{x} = -9P \cos 3t - 9Q \sin 3t$$

$$(-9P \cos 3t - 9Q \sin 3t) + 6(-3P \cos 3t + 3Q \sin 3t) + 9(P \cos 3t + Q \sin 3t) = \cos 3t$$

$$-9P + 18Q + 9P = 1 \text{ and } -9Q - 18P + 9Q = 0$$

$$p = 0 \quad \text{and} \quad Q = \frac{1}{18}$$

$$x = (A + Bt)e^{-3t} + \frac{1}{18} \sin 3t$$

M1**A1****B1****M1****M1****M1****A1****A1**



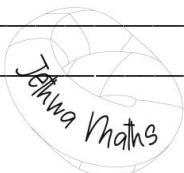
1.

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 2e^{-t}$$

Given that $x = 0$ and $\frac{dx}{dt} = 2$ at $t = 0$,

Find x in terms of t .

(8)



Solutions

1.

$$m^2 + 5m + 6 \quad m = -3, -2$$

M1**A1**

$$\text{C.F. } x = Ae^{-3t} + Be^{-2t}$$

M1

$$x = ke^{-t} \Rightarrow \frac{dx}{dt} = -ke^{-t} \Rightarrow \frac{d^2x}{dt^2} = ke^{-t}$$

$$\Rightarrow ke^{-t} + 5(-ke^{-t}) + 6(ke^{-t}) = 2e^{-t}$$

$$k = 1$$

A1

$$\text{P.I. } x = e^{-t}$$

$$x = Ae^{-3t} + Be^{-2t} + e^{-t}$$

M1

$$\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t} - e^{-t}$$

M1

$$0 = A + B + 1$$

M1

$$2 = -3A - 2B - 1$$

$$A = -1, B = 0$$

$$\text{So, } x = e^{-3t} + e^{-t}$$

A1