



## Solutions

1.

$$\sin x \frac{dy}{dx} - y \cos x = \sin 2x \sin x$$

$$\frac{dy}{dx} - \frac{y \cos x}{\sin x} = \frac{\sin 2x \sin x}{\sin x}$$

$$\frac{dy}{dx} - \frac{y \cos x}{\sin x} = \sin 2x$$

$$\text{Integrating factor} = e^{\int -\frac{\cos x}{\sin x}} = e^{-\ln \sin x}$$

$$= \frac{1}{\sin x}$$

$$\left(\frac{1}{\sin x}\right) \frac{dy}{dx} - \frac{y \cos x}{\sin^2 x} = \frac{\sin 2x}{\sin x}$$

$$\frac{d}{dx} \left(\frac{y}{\sin x}\right) = \sin 2x \times \frac{1}{\sin x}$$

$$\frac{d}{dx} \left(\frac{y}{\sin x}\right) = 2 \cos x$$

$$\frac{y}{\sin x} = \int 2 \cos x \, dx$$

$$y = 2 \sin^2 x + K \sin x$$

**M1**

**M1**

**A1**

**A1**

**M1**

**A1**

**A1**



# Further Maths A-Level Starter Activity



**Topic: Methods in Differential Equations (2)**  
Chapter Reference: Core Pure 2, Chapter 7

**8  
minutes**

1. Find the general solution of the differential equation  $\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + 4x = 0$ , where  $k$  is a real constant, in each of the following cases,

a.  $|k| > 2$  (4)

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b.  $|k| < 2$  (2)

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c.  $|k| = 2$  (2)

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## Solutions

1a.

$m^2 + 2km + 4 = 0$ $\Rightarrow m - k \pm \sqrt{k^2 - 4}$ $x = e^{-kt} \left( Ae^{\sqrt{k^2-4}t} + Be^{-\sqrt{k^2-4}t} \right)$	<b>M1</b> <b>A1</b> <b>M1</b> <b>A1</b>
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1b.

$x = e^{-kt} \left( Ae^{i\sqrt{4-k^2}t} + Be^{-i\sqrt{4-k^2}t} \right)$ $x = e^{-kt} \left( A' \cos \sqrt{4-k^2}t + B' \sin \sqrt{4-k^2}t \right)$	<b>M1</b> <b>A1</b>
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1c.

$x = e^{-2t}(A'' + B''t)$	<b>M1</b> <b>A1</b>
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## Solutions

1a.

$y = \lambda t^2 e^{3t}$ $\frac{dy}{dt} = 2\lambda t e^{3t} + 3\lambda t^2 e^{3t}$ $\frac{d^2y}{dt^2} = 2\lambda e^{3t} + 6\lambda t e^{3t} + 6\lambda t e^{3t} + 9\lambda t^2 e^{3t}$ $2\lambda e^{3t} + 6\lambda t e^{3t} + 6\lambda t e^{3t} + 9\lambda t^2 e^{3t} - 12\lambda t e^{3t} - 18\lambda t^2 e^{3t} + 9\lambda t^2 e^{3t} = 6e^{3t}$ $\lambda = 3$	<b>M1A1</b>  <b>A1</b>  <b>M1</b> <b>A1</b>
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1b.

$m^2 - 6m + 9 = 0$ $(m - 3)^2 = 0$ C.F. $y = (A + 3t)e^{3t}$ G.S. $y = (A + 3t)e^{3t} + 3t^2 e^{3t}$	<b>M1A1</b>  <b>A1</b>
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## Solutions

1.

$m^2 + 6m + 9 = 0$	$m = -3$	<b>M1</b>
C.F. $x = (A + Bt)e^{-3t}$		<b>A1</b>
P.I. $x = P \cos 3t + Q \sin 3t$		<b>B1</b>
$\dot{x} = -3P \cos 3t + 3Q \sin 3t$		<b>M1</b>
$\ddot{x} = -9P \cos 3t - 9Q \sin 3t$		
$(-9P \cos 3t - 9Q \sin 3t) + 6(-3P \cos 3t + 3Q \sin 3t) + 9(P \cos 3t + Q \sin 3t) = \cos 3t$		<b>M1</b>
$-9P + 18Q + 9P = 1$ and $-9Q - 18P + 9Q = 0$		<b>M1</b>
	$p = 0$ and $Q = \frac{1}{18}$	
		<b>A1</b>
$x = (A + Bt)e^{-3t} + \frac{1}{18} \sin 3t$		<b>A1</b>







## Solutions

1.

$m^2 + 5m + 6$	$m = -3, -2$	<b>M1</b>
C.F. $x = Ae^{-3t} + Be^{-2t}$		<b>A1</b>
$x = ke^{-t} \Rightarrow \frac{dx}{dt} = -ke^{-t} \Rightarrow \frac{d^2x}{dt^2} = ke^{-t}$		<b>M1</b>
$\Rightarrow ke^{-t} + 5(-ke^{-t}) + 6(ke^{-t}) = 2e^{-t}$		
$k = 1$		<b>A1</b>
P.I. $x = e^{-t}$		
$x = Ae^{-3t} + Be^{-2t} + e^{-t}$		<b>M1</b>
$\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t} - e^{-t}$		<b>M1</b>
$0 = A + B + 1$		<b>M1</b>
$2 = -3A - 2B - 1$		
$A = -1, B = 0$		
So, $x = e^{-3t} + e^{-t}$		<b>A1</b>

