Topic: Linear Transformations (1)

Chapter Reference: Core Pure 1, Chapter 7

7 minutes

1. The rectangle R has vertices at the points (0,0), (1,0), (1,2) and (0,2).

(a) Find the coordinates of the vertices of the image of R under the transformation given by the matrix

$\mathbf{A} = \begin{pmatrix} a & 4 \\ -1 & 1 \end{pmatrix}$, where a is a constant. **(3)**

(b) Find det **A**, giving your answer in terms of a. **(1)**

Given that the area of the image of R is 18,

(c) Find the value of a. **(3)**

Solutions

1a.

$\begin{pmatrix} a & 4 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 \\ 2 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 8 & a & a+8 \\ 2 & -1 & 1 \end{pmatrix}$	**M1** **A1** **A1**

1b.

det **A** = $a - (-4) = a + 4$	**B1**

1c.

Area of R = 2 Area of R' = 18 Area scale factor is 9 = $a + 4$ $\therefore\ a = 5$	**B1** **M1** **A1**

Further Maths A-Level Starter Activity

Topic: Linear Transformation (2)

Chapter Reference: Core Pure 1, Chapter 7

9 minutes

1. (a) Write down the matrix, $\boldsymbol{A}$, that represents an enlargement, centre (0,0), with scale factor $\sqrt{2}$ **(1)**

(b) The matrix $\boldsymbol{B}$ is given by $\boldsymbol{B} = \begin{pmatrix} \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \\ -\frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{pmatrix}$. Describe fully the geometrical transformation represented by $\boldsymbol{B}$. **(3)**

(c) Given that $\boldsymbol{C} = \boldsymbol{AB}$, show that $\boldsymbol{C} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ **(1)**

(d) Draw a diagram showing the unit square and its image under the transformation represented by $\boldsymbol{C}$. **(2)**

(e) Write down the determinant of $\boldsymbol{C}$ and explain briefly how this value relates to the transformation represented by $\boldsymbol{C}$. **(2)**

Solutions

1a.

$\begin{pmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{pmatrix}$	**B1**

1b.

Rotation (centre O), 45° clockwise	**B1** **B1** **B1**

1c.

Matrix multiplication	**B1**

1d.

$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$	**M1** **A1**

1e.

Det ***C*** = 2 Area of square has been doubled	**B1** **B1**

Further Maths A-Level Starter Activity

Topic: Linear Transformations (3)

Chapter Reference: Core Pure 1, Chapter 7

7 minutes

1. The transformation U, represented by the 2×2 matrix $\boldsymbol{P}$, is a rotation through 90° anticlockwise about the origin.

(a) Write down the matrix $\boldsymbol{P}$. **(1)**

The transformation V, represented by the 2×2 matrix $\boldsymbol{Q}$, is a reflection in the line $y = -x$.

(b) Write down the matrix $\boldsymbol{Q}$. **(1)**

Given that U followed by V is transformation T, which is represented by the matrix $\boldsymbol{R}$,

(c) Express $\boldsymbol{R}$ in terms of $\boldsymbol{P}$ and $\boldsymbol{Q}$, **(1)**

(d) Find the matrix $\boldsymbol{R}$, **(2)**

(e) Give full geometrical description of T as a single transformation. **(2)**

Solutions

1a.

$\boldsymbol{P} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	**B1**

1b.

$\boldsymbol{Q} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	**B1**

1c.

$\boldsymbol{R} = \boldsymbol{QP}$	**B1**

1d.

$\boldsymbol{R} = \begin{pmatrix} \mathbf{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$	**M1** **M1**

1e.

Reflection in the y-axis	**A1** **A1**

Topic: Linear Transformations (4) Chapter Reference: Core Pure 1, Chapter 7	9 minutes

1. The transformation R is represented by the matrix $\boldsymbol{M}$, where $\boldsymbol{M} = \begin{pmatrix} 3 & a & 0 \\ 2 & b & 0 \\ c & 0 & 1 \end{pmatrix}$ and where a, b, and c are constants.

Given that $\boldsymbol{M} = \boldsymbol{M}^{-1}$,

(a) Find the values of a, b and c **(5)**

(b) Evaluate the determinant of $\boldsymbol{M}$ **(2)**

(c) Find an equation satisfied by all the points which remain invariant under R **(2)**

Solutions

1a.

$\boldsymbol{MM}^{-1} = \boldsymbol{I}$ $9 + 2a = 1 \Rightarrow a = -4$ $6 + 2b = 0 \Rightarrow b = -3$ $4c = 0 \Rightarrow c = 0$	**M1** **M1** **M1** **A1** **A1**

1b.

det $\boldsymbol{M} = -1$	**M1** **A1**

1c.

$x = 2y$	**A1** **A1**

Further Maths A-Level Starter Activity

Topic: Linear Transformations (5) Chapter Reference: Core Pure 1, Chapter 7	10 minutes

1. $A = \begin{pmatrix} 3\sqrt{2} & 0 \\ 0 & 3\sqrt{2} \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $C = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$

It is given that the matrix ***D*** = ***CA***, and that the matrix ***E*** = ***DB***.

(a) Find ***D***. **(2)**

(b) Show that ***E*** = $\begin{pmatrix} -3 & 3 \\ 3 & 3 \end{pmatrix}$ **(1)**

The triangle *ORS* has vertices at the points with coordinates (0,0), (-15, 15) and (4, 21). This triangle is transformed onto the triangle *OR'S'* by the transformation described by E.

(c) Find the coordinates of the vertices of triangle *OR'S'*. **(4)**

(d) Find the area of triangle *OR'S'* and deduce the area of triangle *ORS*. **(3)**

Solutions

1a.

$\begin{pmatrix} 3 & -3 \\ 3 & 3 \end{pmatrix}$	**M1** **A1**

1b.

Part (a) × $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	**B1**

1c.

$\begin{pmatrix} -3 & 3 \\ 3 & 3 \end{pmatrix}\begin{pmatrix} 0 & -15 & 4 \\ 0 & 15 & 21 \end{pmatrix} = \begin{pmatrix} 0 & 90 & 51 \\ 0 & 0 & 75 \end{pmatrix}$ so (0,0), (90,0) and (51, 75)	**M1** **A1** **A1** **A1**

1d.

Area of $\Delta\ OR'S'$ is $\frac{1}{2} \times 90 \times 75 = 3375$ Determinant of E is −18 or use area scale factor of enlargement So area of $\Delta\ OR'S'$ is 3375 ÷ 18 = 187.5	**B1** **M1** **A1**