

**Further Maths
A-Level Starter
Activity**



Topic: Linear Transformations (1)

Chapter Reference: Core Pure 1, Chapter 7

**7
minutes**

1. The rectangle R has vertices at the points $(0,0)$, $(1,0)$, $(1,2)$ and $(0,2)$.

(a) Find the coordinates of the vertices of the image of R under the transformation given by the matrix

$\mathbf{A} = \begin{pmatrix} a & 4 \\ -1 & 1 \end{pmatrix}$, where a is a constant. **(3)**

(b) Find $\det \mathbf{A}$, giving your answer in terms of a . **(1)**

Given that the area of the image of R is 18,

(c) Find the value of a . **(3)**



Solutions

1a.

$\begin{pmatrix} a & 4 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 \\ 2 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 8 & a & a+8 \\ 2 & -1 & 1 \end{pmatrix}$	M1 A1 A1
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1b.

$\det \mathbf{A} = a - (-4) = a + 4$	B1
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1c.

Area of $R = 2$ Area of $R' = 18$ Area scale factor is $9 = a + 4$ $\therefore a = 5$	B1 M1 A1
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1. (a) Write down the matrix, A , that represents an enlargement, centre $(0,0)$, with scale factor $\sqrt{2}$ (1)

- (b) The matrix B is given by $B = \begin{pmatrix} \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \\ -\frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{pmatrix}$. Describe fully the geometrical transformation represented by B . (3)

- (c) Given that $C = AB$, show that $C = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ (1)

- (d) Draw a diagram showing the unit square and its image under the transformation represented by C . (2)

- (e) Write down the determinant of C and explain briefly how this value relates to the transformation represented by C . (2)



Solutions

1a.

$\begin{pmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{pmatrix}$	B1
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1b.

Rotation (centre O), 45° clockwise	B1 B1 B1
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1c.

Matrix multiplication	B1
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1d.

$\begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix}$	M1 A1
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1e.

Det $C = 2$ Area of square has been doubled	B1 B1
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1. The transformation U , represented by the 2×2 matrix P , is a rotation through 90° anticlockwise about the origin.

(a) Write down the matrix P . (1)

The transformation V , represented by the 2×2 matrix Q , is a reflection in the line $y = -x$.

(b) Write down the matrix Q . (1)

Given that U followed by V is transformation T , which is represented by the matrix R ,

(c) Express R in terms of P and Q , (1)

(d) Find the matrix R , (2)

(e) Give full geometrical description of T as a single transformation. (2)



Solutions

1a.

$P = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	B1
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1b.

$Q = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	B1
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1c.

$R = QP$	B1
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1d.

$R = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	M1 M1
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1e.

Reflection in the y-axis	A1 A1
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1. The transformation R is represented by the matrix M , where $M = \begin{pmatrix} 3 & a & 0 \\ 2 & b & 0 \\ c & 0 & 1 \end{pmatrix}$ and where a , b , and c are constants.

Given that $M = M^{-1}$,

(a) Find the values of a , b and c (5)

(b) Evaluate the determinant of M (2)

(c) Find an equation satisfied by all the points which remain invariant under R (2)



Solutions

1a.

$MM^{-1} = I$	
$9 + 2a = 1 \Rightarrow a = -4$	M1
$6 + 2b = 0 \Rightarrow b = -3$	M1
$4c = 0 \Rightarrow c = 0$	M1
	A1
	A1

1b.

$\det M = -1$	
	M1
	A1

1c.

$x = 2y$	
	A1
	A1

Further Maths
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Topic: Linear Transformations (5)

Chapter Reference: Core Pure 1, Chapter 7

**10
minutes**

1. $A = \begin{pmatrix} 3\sqrt{2} & 0 \\ 0 & 3\sqrt{2} \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $C = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$

It is given that the matrix $D = CA$, and that the matrix $E = DB$.

(a) Find D .

(2)

(b) Show that $E = \begin{pmatrix} -3 & 3 \\ 3 & 3 \end{pmatrix}$

(1)

(c) Find the coordinates of the vertices of triangle $OR'S'$.

(4)

(d) Find the area of triangle $OR'S'$ and deduce the area of triangle ORS .

(3)



Solutions

1a.

$\begin{pmatrix} 3 & -3 \\ 3 & 3 \end{pmatrix}$	M1 A1
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1b.

Part (a) $\times \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	B1
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1c.

$\begin{pmatrix} -3 & 3 \\ 3 & 3 \end{pmatrix} \begin{pmatrix} 0 & -15 & 4 \\ 0 & 15 & 21 \end{pmatrix} = \begin{pmatrix} 0 & 90 & 51 \\ 0 & 0 & 75 \end{pmatrix}$ so (0,0), (90,0) and (51, 75)	M1 A1 A1 A1
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1d.

Area of $\Delta OR'S'$ is $\frac{1}{2} \times 90 \times 75 = 3375$ Determinant of E is -18 or use area scale factor of enlargement So area of $\Delta OR'S'$ is $3375 \div 18 = 187.5$	B1 M1 A1
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