A-Level Starter Activity

Jethwa Maths

Topic: Circles and Lines Chapter Reference: Pure 1, Chapter 6	10 minutes

1. A circle has equation $(x - 5)^2 + (y + 2)^2 = 25$.

a. Find the coordinates of the centre and the length of the diameter. **(2)**

b. Find the equation of the line which passes through C and the point P (7, 2) **(3)**

c. Calculate the length of CP and hence determine whether P lies inside or outside the circle. **(2)**

d. Determine algebraically whether the line with equation $y = 2x$ meets the circle. **(4)**

Solutions

1a.

Centre *C* (5, -2)	**M1**
Radius = $\sqrt{25}$ = 5 Therefore diameter = 10	**M1**

1b.

Gradient *m* = $\frac{2--2}{7-5} = 2$	**M1**
Equation of the line is: $y - 2 = 2(x - 7)$ $y - 2 = 2x - 14$	**M1**
$y = 2x - 12$	**M1**

1c.

$CP = \sqrt{(7-5)^2 + (2--2)^2}$	**M1**
$= \sqrt{20} = 2\sqrt{5}$	**M1**

1d.

$y = 2x$ $(x - 5)^2 + (2x + 2)^2 = 25$	**M1**
$x^2 - 10x + 25 + 4x^2 + 8x + 4 = 25$ $5x^2 - 2x + 4 = 0$	**M1**
Discriminant: $(-2)^2 - 4(5)(4) = -76$ as discriminant is less than 0	**M1**
There are no real roots and therefore line does not intersect.	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Circles and tangents Chapter Reference: Pure 1, Chapter 6	8 minutes

1. The point A and B have coordinates (-2, 11) and (8, 1) respectively. Given that AB is a diameter of the circle C

a. Show that the centre of C has coordinates (3, 6) **(1)**

b. Find an equation for C. **(3)**

c. Verify that the point (10, 7) lies on C. **(2)**

d. Find an equation of the tangent to C at the point (10, 7), giving your answer in the form $y = mx + c$, where m and c are constants. **(4)**

Solutions

1a.

Centre: $(\frac{-2+8}{2}, \frac{11+1}{2})$ = (3, 6)	**M1**

1b.

$r^2 = (8 - 3)^2 + (6 - 1)^2 = 50$	**M1**
Equation of *C* is, $(x - 3)^2 + (y - 6)^2 = 50$	**M1 M1**

1c.

When $x = 10$, $y = 7$ $(10 - 3)^2 + (7 - 6)^2 = 7^2 + 1^2 = 50$	**M1**
Therefore (10, 7) lies on *C*.	**M1**

1d.

Gradient of $MP = \frac{7-6}{10-3} = \frac{1}{7}$	**M1**
Gradient of tangent = -7	**M1**
Equation of tangent is: $y - 7 = -7(x - 10)$ $y - 7 = -7x + 70$	**M1**
$y = -7x + 77$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Equation of a Circle

Chapter Reference: Pure 1, Chapter 6

8 minutes

1. A circle C has equation,

$$x^2 + y^2 - 4x + 10y = k$$

Where k is a constant.

a. Find the coordinates of the centre of C **(2)**

b. State the range of possible vales for k **(2)**

2. A circle has centre (-1, 7) and passes through the point (0, 0). Find an equation for C. **(3)**

Solutions

1a.

$(x-2)^2 - 4 + (y+5)^2 - 25 = k$ $(x-2)^2 + (y+5)^2 = k + 29$	**M1**
Centre at (2, -5)	**M1**

1b.

$r^2 = k + 29$ As $r > 0$	**M1**
$k + 29 > 0$ $k > -29$	**M1**

2.

$r^2 = (-1)^2 + (7)^2 = 50$	**M1**
Equation of C: $(x - -1)^2 + (y - 7)^2 = 50$	**M1**
$(x + 1)^2 + (y - 7)^2 = 50$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Circles Exam Question Chapter Reference: Pure 1, Chapter 6	8 minutes

1. The circle C has equation $x^2 + y^2 - 20x - 24y + 195 = 0$.
The centre of C is at the point M,
a. Find the coordinates of the point M **(3)**
b. Find the radius of the circle C **(1)**

N is the point with coordinates (25, 32).
c. Find the length of the line MN. **(2)**

The tangent to C at a point P on the circle passes through point N.
d. Find the length of the line NP. **(2)**

Solutions

1a.

$x^2 + y^2 - 20x - 24y + 195 = 0$ $(x - 10)^2 - 100 + (y - 12)^2 - 144 + 195 = 0$	**M1**
$(x - 10)^2 + (y - 12)^2 = 49$	**M1**
Centre: $M = (10, 12)$	**M1**

1b.

Radius = $\sqrt{49} = 7$	**M1**

1c.

$MN = \sqrt{(25 - 10)^2 + (32 - 12)^2}$	**M1**
$= \sqrt{625} = 25$	**M1**

1d.

$NP = \sqrt{(25)^2 - (7)^2}$	**M1**
$= \sqrt{576} = 24$	**M1**