Further Maths A-Level Starter Activity

Topic: Matrices (1)

Chapter Reference: Core Pure 1, Chapter 6

10 minutes

1. The matrix **M** is defined by

$$\mathbf{M} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

(a) Find the matrix

(i) $\mathbf{M}^2$ **(3)**

(ii) $\mathbf{M}^4$ **(1)**

2. The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

(a) Calculate:

(i) **A** + **B** **(3)**

(ii) **BA** **(2)**

(iii) **AB** **(1)**

Solutions

1.

$\mathbf{M}^2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$	**M1 A2**
$\mathbf{M}^4 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$	**B1**

2.

$\mathbf{A} + \mathbf{B} = \begin{bmatrix} \sqrt{3} & 0 \\ 1 & 0 \end{bmatrix}$	**M1 A1**
$\mathbf{BA} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	**B3**
$\mathbf{AB} = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{-1}{2} \end{bmatrix}$	**B1**

Further Maths A-Level Starter Activity

Topic: Matrices (2)

Chapter Reference: Pure Core 1, Chapter 6

9 minutes

1. The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 5 & 0 \\ 0 & 2 \end{pmatrix}$ and I is the 2 × 2 identity matrix.

Find the values of the constants a and b for which $a\mathbf{A} + b\mathbf{B} = \mathbf{I}$ **(4)**

2. The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} a & 2 \\ 3 & 4 \end{pmatrix}$ and **I** is the 2 × 2 identity matrix.

a) Find $\mathbf{A} - 4\mathbf{I}$. **(2)**

b) Given that **A** is singular, find the value of a. **(3)**

Solutions

1.

$3a + 5b = 1$, $a + 2b = 1$	**M1** **M1**
Solving simultaneously	**M1**
$a = -3$, $b = 2$	**A1**

2a.

$\begin{pmatrix} a-4 & 2 \\ 3 & 0 \end{pmatrix}$	**B1** **B1**

2b.

$4a - 6$	**B1** **B1**
$a = \frac{3}{2}$	**A1**

Further Maths A-Level Starter Activity

Topic: Matrices (3)

Chapter Reference: Core Pure 1, Chapter 6

9 minutes

1. The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix}$.

(i) Find the value of the determinant of **M**. **(3)**

(ii) State, giving a brief reason, whether **M** is singular or non-singular. **(1)**

2. The matrix **C** is given by $\mathbf{C} = \begin{pmatrix} 1 & 2 \\ 3 & 8 \end{pmatrix}$.

(i) Given that **C** = **AB**, where A = $\begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$, find $\mathbf{B}^{-1}$. **(5)**

Solutions

1a.

$2\begin{bmatrix}2 & 1\\1 & 3\end{bmatrix} - 1\begin{bmatrix}1 & 1\\1 & 3\end{bmatrix} + 3\begin{bmatrix}1 & 2\\1 & 1\end{bmatrix}$	**M1**
2 x 5 – 1 x 2 + 3 x – 1	**A1**
5	**A1**

1b.

M is non-singular as det **M** non-zero	**B1**

2.

B = $\mathbf{A}^{-1}\mathbf{C}$ $\frac{1}{2}\begin{pmatrix}14 & 2\\-5 & 0\end{pmatrix}$	**B1** **M1** **M1** **A1** **A1**

Topic: Matrices (4)

Chapter Reference: Core Pure 1, Chapter 6

7 minutes

1. The matrix **D** is given by $\mathbf{D} = \begin{pmatrix} a & 2 & 0 \\ 3 & 1 & 2 \\ 0 & -1 & 1 \end{pmatrix}$, where $a \neq 2$.

   (i) Find $\mathbf{D}^{-1}$. **(7)**

Solutions

1.

$\Delta = \det \mathbf{D} = 3a - 6$ $\mathbf{D}^{-1} = \frac{1}{\Delta}\begin{pmatrix} 3 & -2 & 4 \\ -3 & a & -2a \\ -3 & a & a-6 \end{pmatrix}$	**M1** **M1** **A1** **M1** **A1** **B1** **A1**

Further Maths A-Level Starter Activity

Topic: Matrices (5)

Chapter Reference: Core Pure 1, Chapter 6

10 minutes

1. The matrix **B** is given by $\mathbf{B} = \begin{pmatrix} a & 1 & 3 \\ 2 & 1 & -1 \\ 0 & 1 & 2 \end{pmatrix}$

(i) Given that **B** is singular, show that $a = -\frac{2}{3}$. **(3)**

(ii) Given instead that **B** is non-singular, find the inverse matrix $\mathbf{B}^{-1}$. **(4)**

(iii) Hence, or otherwise, solve the equations

$$-x + y + 3z = 1,$$
$$2x + y - z = 4,$$
$$y + 2z = -1$$

(3)

Solutions

1.

det(B) = 0 $3a - 4 + 6 = 0$ $a = -\frac{2}{3}$	**M1** **A1 A1**
$\frac{1}{3a+2}\begin{pmatrix} 3 & 1 & -4 \\ -4 & 2a & a+6 \\ 2 & -a & a-2 \end{pmatrix}$	**M1** **A1** **B1** **A1**
$a = -1$ $\begin{pmatrix} -11 \\ 17 \\ -9 \end{pmatrix}$ $x = -11$, $y = 17$, $z = -9$	**M1** **M1** **A1**