



1. The matrix **M** is defined by

$$\mathbf{M} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

(a) Find the matrix

(i)  $\mathbf{M}^2$  (3)

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(ii)  $\mathbf{M}^4$  (1)

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2. The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

(a) Calculate:

(i)  $\mathbf{A} + \mathbf{B}$  (3)

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(ii)  $\mathbf{BA}$  (2)

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(iii)  $\mathbf{AB}$  (1)

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## Solutions

1.

$\mathbf{M}^2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$	<b>M1 A2</b>
$\mathbf{M}^4 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$	<b>B1</b>

2.

$\mathbf{A} + \mathbf{B} = \begin{bmatrix} \sqrt{3} & 0 \\ 1 & 0 \end{bmatrix}$	<b>M1 A1</b>
$\mathbf{BA} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	<b>B3</b>
$\mathbf{AB} = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{-1}{2} \end{bmatrix}$	<b>B1</b>





1. The matrices **A** and **B** are given by  $\mathbf{A} = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$  and  $\mathbf{B} = \begin{pmatrix} 5 & 0 \\ 0 & 2 \end{pmatrix}$  and **I** is the  $2 \times 2$  identity matrix.

Find the values of the constants  $a$  and  $b$  for which  $a\mathbf{A} + b\mathbf{B} = \mathbf{I}$  (4)

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2. The matrix **A** is given by  $\mathbf{A} = \begin{pmatrix} a & 2 \\ 3 & 4 \end{pmatrix}$  and **I** is the  $2 \times 2$  identity matrix.

a) Find  $\mathbf{A} - 4\mathbf{I}$ . (2)

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b) Given that **A** is singular, find the value of  $a$ . (3)

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## Solutions

1.

$3a + 5b = 1,$	<b>M1</b>
$a + 2b = 1$	<b>M1</b>
Solving simultaneously	<b>M1</b>
$a = -3, b = 2$	<b>A1</b>

2a.

$\begin{pmatrix} a - 4 & 2 \\ 3 & 0 \end{pmatrix}$	<b>B1</b>
	<b>B1</b>

2b.

$4a - 6$	<b>B1</b>
	<b>B1</b>
$a = \frac{3}{2}$	<b>A1</b>



Further Maths  
A-Level Starter  
Activity



**Topic: Matrices (3)**

Chapter Reference: Core Pure 1, Chapter 6

**9**  
**minutes**

1. The matrix  $\mathbf{M}$  is given by  $\mathbf{M} = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix}$ .

- (i) Find the value of the determinant of  $\mathbf{M}$ . (3)

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- (ii) State, giving a brief reason, whether  $\mathbf{M}$  is singular or non-singular. (1)

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2. The matrix  $\mathbf{C}$  is given by  $\mathbf{C} = \begin{pmatrix} 1 & 2 \\ 3 & 8 \end{pmatrix}$ .

- (i) Given that  $\mathbf{C} = \mathbf{AB}$ , where  $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$ , find  $\mathbf{B}^{-1}$ . (5)

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## Solutions

1a.

$2\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} - 1\begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix} + 3\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$	<b>M1</b>
$2 \times 5 - 1 \times 2 + 3 \times -1$	<b>A1</b>
5	<b>A1</b>

1b.

<b>M</b> is non-singular as $\det \mathbf{M}$ non-zero	<b>B1</b>
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2.

$\mathbf{B} = \mathbf{A}^{-1}\mathbf{C}$	<b>B1</b>
$\frac{1}{2} \begin{pmatrix} 14 & 2 \\ -5 & 0 \end{pmatrix}$	<b>M1</b>
	<b>M1</b>
	<b>A1</b>
	<b>A1</b>





## Solutions

1.

$$\Delta = \det \mathbf{D} = 3a - 6$$

$$\mathbf{D}^{-1} = \frac{1}{\Delta} \begin{pmatrix} 3 & -2 & 4 \\ -3 & a & -2a \\ -3 & a & a - 6 \end{pmatrix}$$

**M1**

**M1**

**A1**

**M1**

**A1**

**B1**

**A1**







1. The matrix **B** is given by  $\mathbf{B} = \begin{pmatrix} a & 1 & 3 \\ 2 & 1 & -1 \\ 0 & 1 & 2 \end{pmatrix}$

(i) Given that **B** is singular, show that  $a = -\frac{2}{3}$ . (3)

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(ii) Given instead that **B** is non-singular, find the inverse matrix  $\mathbf{B}^{-1}$ . (4)

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(iii) Hence, or otherwise, solve the equations

$$-x + y + 3z = 1,$$

$$2x + y - z = 4,$$

$$y + 2z = -1$$

(3)

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## Solutions

1.

$\det(B) = 0$ $3a - 4 + 6 = 0$ $a = -\frac{2}{3}$	<b>M1</b> <b>A1 A1</b>
$\frac{1}{3a+2} \begin{pmatrix} 3 & 1 & -4 \\ -4 & 2a & a+6 \\ 2 & -a & a-2 \end{pmatrix}$	<b>M1</b> <b>A1</b> <b>B1</b> <b>A1</b>
$a = -1$ $\begin{pmatrix} -11 \\ 17 \\ -9 \end{pmatrix}$ $x = -11, y = 17, z = -9$	<b>M1</b> <b>M1</b> <b>A1</b>

