Further Maths A-Level Starter Activity

Topic: Polar Coordinates and Equations (1) Chapter Reference: Core Pure 2, Chapter 5	10 minutes

1. A circle C_1 has cartesian equation $x^2 + (y-6)^2 = 36$. Show that the polar equation of C_1 is $r = 12\sin\theta$ **(4)**

2. A curve has polar equation $r(1-\sin\theta) = 4$. Find its cartesian equation in the form $y = f(x)$. **(6)**

Solutions

1.

$x^2 + y^2 - 12y + 36 = 36$ Use of $y = r\sin\theta$ ($x = r\cos\theta$ PI)	**M1**
Use of $x^2 + y^2 = r^2$	**M1**
$r^2 - 12r\sin\theta + 36 = 36$	**m1**
$\Rightarrow r = 12\sin\theta$ **CSO AG**	**A1**

2.

$r - r\sin\theta = 4$	**M1**
$r - y = 4$ $r\sin\theta = y$ stated or used	**B1**
$r = y + 4$	**A1**
$x^2 + y^2 = (y+4)^2$ $r^2 = x^2 + y^2$ used	**M1**
$x^2 + y^2 = y^2 + 8y + 16$ ft one slip	**A1F**
$y = \dfrac{x^2 - 16}{8}$	**A1**

Topic: Polar Coordinates and Equations (2)
Chapter Reference: Core Pure 2, Chapter 5

10 minutes

1. A circle with cartesian equation $x^2 + y^2 = 9$ intersects the curve $r = 6(1 - \cos\theta)$, $0 \leq \theta < 2\pi$ at the points A and B. Find the polar coordinates of A and B. **(4)**

2. a. Show that $(\cos\theta + \sin\theta)^2 = 1 + \sin 2\theta$. **(1)**

   b. A curve has cartesian equation $(x^2 + y^2)^3 = (x + y)^4$. Given that $r \geq 0$, show that the polar equation of the curve is $r = 1 + \sin 2\theta$. **(4)**

Solutions

1.

$x^2+y^2=9 \Rightarrow r^2=9$ PI	**B1**
A & B: $3=6-6\cos\theta \Rightarrow \cos\theta=\frac{1}{2}$	**M1**
Pts of intersection $\left(3,\frac{\pi}{3}\right)$; $\left(3,\frac{5\pi}{3}\right)$ OE (accept 'different' values of θ not in the given interval)	**A2**

2a.

$(\cos\theta+\sin\theta)^2=\cos^2\theta+\sin^2\theta+2\cos\theta\sin\theta$ $=1+\sin 2\theta$ AG (be convinced)	**B1**

2b.

$(x^2+y^2)^3=(x+y)^4$ $(r^2)^3=(r\cos\theta+r\sin\theta)^4$ [M1 for one of $x^2+y^2=r^2$ OE, $x=r\cos\theta$, $y=r\sin\theta$ used]	**M2,1,0**
$r^6=r^4(\cos\theta+\sin\theta)^4$ $r^6=r^4(1+\sin 2\theta)^2$ $r^2=(1+\sin 2\theta)^2$ Uses (a) OE at any stage	**M1**
$\Rightarrow r=(1+\sin 2\theta)$ $\{r\geq 0\}$ CSO; AG	**A1**

Further Maths A-Level Starter Activity

Topic: Sketching Curves (3) Chapter Reference: Core Pure 2, Chapter 5	10 minutes

1. $r = e^{-2\theta}$, for $0 \leq \theta \leq \pi$.

   Sketch the curve, stating the polar coordinates of the point at which r takes its greatest value. **(2)**

2. Sketch the curve with equation $r = 1 - sin2\theta$, for $0 \leq \theta \leq 2\pi$.

   State the maximum value of r and the corresponding values of θ. **(4)**

3. $r = \sqrt{3} + \tan\theta$, for $-\frac{1}{3}\pi \leq \theta \leq \frac{1}{4}\pi$.

   a. State the greatest value of r and the corresponding value of θ. **(2)**

   b. Sketch the curve. **(2)**

Solutions

1.

State $r = 1$ and $\theta = 0$. Clear attempt at differentiation	**B1**
Get 1.60 Or answer which rounds	**B1**

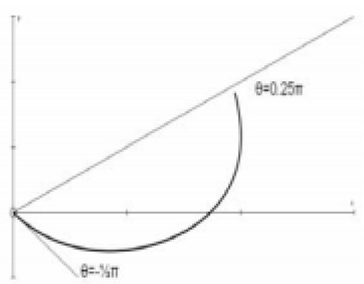
2.

roughly Correct shape; 2 branches only.	**B1**
Clear symmetry in correct rays	**B1**
Get max. $r = 2$	**B1**
At $\theta = \frac{3}{4}\pi$ and $\frac{7}{4}\pi$; both required (allow correct answers not in $0 \le \theta < 2\pi$ here)	**B1**

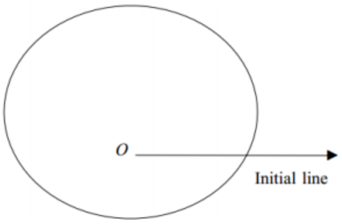
3a.

$r = \sqrt{3} + 1$ when $\theta = \frac{1}{4}\pi$	**B2**

3b.

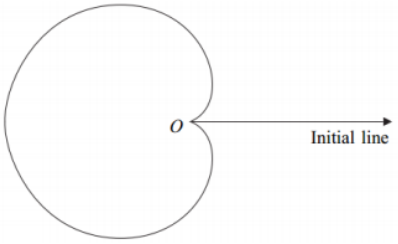
 Correct r at correct end-values of θ; Ignore extra θ used	**B1**
Correct shape with r not decreasing	**B1**

1. A curve C_2 with polar equation $r = 2\sin\theta + 5$, $0 \leq \theta \leq 2\pi$ is shown in the diagram.



Calculate the area bounded by C_2. **(6)**

2. The diagram shows the curve C with polar equation $r = 6(1 - \cos\theta)$, $0 \leq \theta \leq 2\pi$



Find the area of the region bounded by the curve C. **(6)**

Solutions

1.

Area = $\frac{1}{2}\int(2\sin\theta+5)^2\,d\theta$. Use of $\frac{1}{2}\int r^2\,d\theta$.	**M1**
.. = $\frac{1}{2}\int_0^{2\pi}(4\sin^2\theta+20\sin\theta+25)d\theta$ Correct expn. of $(2\sin\theta+5)^2$	**B1**
Correct limits	**B1**
$=\frac{1}{2}\int_0^{2\pi}(2(1-\cos 2\theta)+20\sin\theta+25)d$ Attempt to write $\sin^2\theta$ in terms of $\cos 2\theta$.	**M1**
$=\frac{1}{2}[27\theta-\sin 2\theta-20\cos\theta]_0^{2\pi}$ Correct integration ft wrong coeffs	**A1**
$=27\pi$. CSO	**A1**

2.

Area = $\frac{1}{2}\int 36(1-\cos\theta)^2\,d\theta$ use of $\frac{1}{2}\int r^2\,d\theta$	**M1**
…= $\frac{1}{2}\int_0^{2\pi}36(1-2\cos\theta+\cos^2\theta)\,d\theta$ for correct explanation of $[6(1-\cos\theta)]^2$	**B1**
for correct limits	**B1**
$=9\int_0^{2\pi}2-4\cos\theta+(\cos 2\theta+1)\,d\theta$ Attempt to write $\cos^2\theta$ in terms of $\cos 2\theta$.	**M1**
$=\left[27\theta-36\sin\theta+\frac{9}{2}\sin 2\theta\right]_0^{2\pi}$ Correct integration; only ft if integrating $a+b\cos\theta+c\cos 2\theta$ with non-zero a, b, c.	**A1**
$=54\pi$ CSO	**A1**

Further Maths A-Level Starter Activity

Topic: Tangents to Polar Curves (5)

Chapter Reference: Core Pure 2, Chapter 5

10 minutes

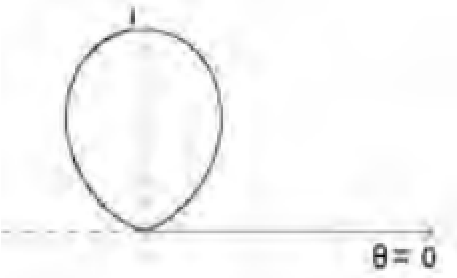
1. The equation of a curve is $r = \sin^3\theta$, for $0 \leq \theta < \pi$. Find the equations of the tangents at the pole and sketch the curve. **(4)**

2. $r = \sqrt{3} + \tan\theta$, for $-\frac{1}{3}\pi \leq \theta \leq \frac{1}{4}\pi$. Find the equation of the tangent at the pole. **(2)**

3. $r = 1 + \cos 2\theta$, for $0 \leq \theta \leq 2\pi$. Find the equations of the tangents at the pole. **(2)**

Solutions

1.

Solve for r=0 for at least one θ θ need not be correct	**M1**
Get (θ) = 0 and π Ignore extra answers out of range	**A1**
 General shape (symmetry stated or approximately seen)	**B1**
Tangents at θ=0, π and max r seen	**B1**

2.

Attempt to solve r=0, tan $\theta = -\sqrt{3}$ Allow $\pm\sqrt{3}$	**M1**
Get $\theta = -\frac{1}{3}\pi$ only Allow $-60°$	**A1**

3.

Solve r = 0 for θ, giving $\theta = \frac{1}{2}\pi$ and $\frac{3}{2}\pi$ Two θ needed (rads only); ignore θ out of range	**M1A1**