

**Further Maths
A-Level Starter
Activity**



Topic: Polar Coordinates and Equations (1)
Chapter Reference: Core Pure 2, Chapter 5

**10
minutes**

1. A circle C_1 has cartesian equation $x^2 + (y - 6)^2 = 36$. Show that the polar equation of C_1 is $r = 12 \sin \theta$ (4)

2. A curve has polar equation $r(1 - \sin \theta) = 4$. Find its cartesian equation in the form $y = f(x)$. (6)



Solutions

1.

$x^2 + y^2 - 12y + 36 = 36$ Use of $y = r \sin \theta$ ($x = r \cos \theta$ PI)	M1
Use of $x^2 + y^2 = r^2$	M1
$r^2 - 12r \sin \theta + 36 = 36$	m1
$\Rightarrow r = 12 \sin \theta$ CSO AG	A1

2.

$r - r \sin \theta = 4$	M1
$r - y = 4$ $r \sin \theta = y$ stated or used	B1
$r = y + 4$	A1
$x^2 + y^2 = (y + 4)^2$ $r^2 = x^2 + y^2$ used	M1
$x^2 + y^2 = y^2 + 8y + 16$ ft one slip	A1F
$y = \frac{x^2 - 16}{8}$	A1



Solutions

1.

$x^2 + y^2 = 9 \Rightarrow r^2 = 9$ PI	B1
$A \ \& \ B: 3 = 6 - 6 \cos \theta \Rightarrow \cos \theta = \frac{1}{2}$	M1
Pts of intersection $\left(3, \frac{\pi}{3}\right); \left(3, \frac{5\pi}{3}\right)$ OE (accept 'different' values of θ not in the given interval)	A2

2a.

$(\cos \theta + \sin \theta)^2 = \cos^2 \theta + \sin^2 \theta + 2 \cos \theta \sin \theta$ $= 1 + \sin 2\theta$ AG (be convinced)	B1
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2b.

$(x^2 + y^2)^3 = (x + y)^4$ $(r^2)^3 = (r \cos \theta + r \sin \theta)^4$ [M1 for one of $x^2 + y^2 = r^2$ OE, $x = r \cos \theta, y = r \sin \theta$ used]	M2,1,0
$r^6 = r^4 (\cos \theta + \sin \theta)^4$ $r^6 = r^4 (1 + \sin 2\theta)^2$	M1
$r^2 = (1 + \sin 2\theta)^2$ Uses (a) OE at any stage	
$\Rightarrow r = (1 + \sin 2\theta) \{r \geq 0\}$ CSO; AG	A1



**Further Maths
A-Level Starter
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Topic: Sketching Curves (3)
Chapter Reference: Core Pure 2, Chapter 5

**10
minutes**

1. $r = e^{-2\theta}$, for $0 \leq \theta \leq \pi$.

Sketch the curve, stating the polar coordinates of the point at which r takes its greatest value. (2)

2. Sketch the curve with equation $r = 1 - \sin 2\theta$, for $0 \leq \theta \leq 2\pi$.

State the maximum value of r and the corresponding values of θ . (4)

3. $r = \sqrt{3} + \tan \theta$, for $-\frac{1}{3}\pi \leq \theta \leq \frac{1}{4}\pi$.

a. State the greatest value of r and the corresponding value of θ . (2)

b. Sketch the curve. (2)

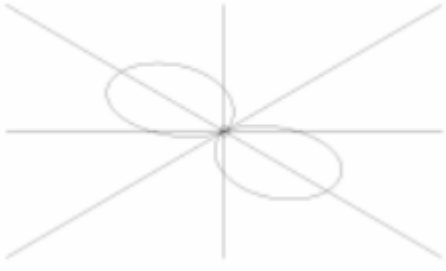


Solutions

1.

State $r=1$ and $\theta=0$. Clear attempt at differentiation	B1
Get 1.60 Or answer which rounds	B1

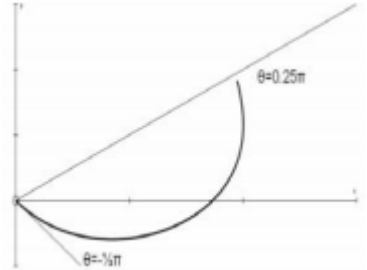
2.

<p>roughly</p>  <p style="text-align: right;">Correct shape; 2 branches only.</p>	B1
Clear symmetry in correct rays	B1
Get max. $r = 2$	B1
At $\theta = \frac{1}{4}\pi$ and $\frac{7}{4}\pi$; both required (allow correct answers not in $0 \leq \theta < 2\pi$ here)	B1

3a.

$r = \sqrt{3} + 1$ when $\theta = \frac{1}{4}\pi$	B2
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3b.

 <p style="text-align: right;">Correct r at correct end-values of θ; Ignore extra θ used</p>	B1
Correct shape with r not decreasing	B1

Solutions

1.

Area = $\frac{1}{2} \int (2 \sin \theta + 5)^2 d\theta$. Use of $\frac{1}{2} \int r^2 d\theta$.	M1
$\therefore = \frac{1}{2} \int_0^{2\pi} (4 \sin^2 \theta + 20 \sin \theta + 25) d\theta$ Correct expn. of $(2 \sin \theta + 5)^2$	B1
Correct limits	B1
$= \frac{1}{2} \int_0^{2\pi} (2(1 - \cos 2\theta) + 20 \sin \theta + 25) d\theta$ Attempt to write $\sin^2 \theta$ in terms of $\cos 2\theta$.	M1
$= \frac{1}{2} [27\theta - \sin 2\theta - 20 \cos \theta]_0^{2\pi}$ Correct integration ft wrong coeffs	A1
$= 27\pi$ CSO	A1

2.

Area = $\frac{1}{2} \int 36(1 - \cos \theta)^2 d\theta$ use of $\frac{1}{2} \int r^2 d\theta$	M1
$\therefore = \frac{1}{2} \int_0^{2\pi} 36(1 - 2 \cos \theta + \cos^2 \theta) d\theta$ for correct explanation of $[6(1 - \cos \theta)]^2$	B1
for correct limits	B1
$= 9 \int_0^{2\pi} 2 - 4 \cos \theta + (\cos 2\theta + 1) d\theta$ Attempt to write $\cos^2 \theta$ in terms of $\cos 2\theta$.	M1
$= \left[27\theta - 36 \sin \theta + \frac{9}{2} \sin 2\theta \right]_0^{2\pi}$ Correct integration; only ft if integrating $a + b \cos \theta + c \cos 2\theta$ with non-zero a, b, c .	A1
$= 54\pi$ CSO	A1



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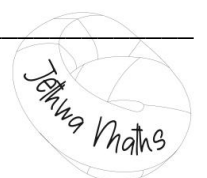
Topic: Tangents to Polar Curves (5)
Chapter Reference: Core Pure 2, Chapter 5

**10
minutes**

1. The equation of a curve is $r = \sin^3 \theta$, for $0 \leq \theta < \pi$. Find the equations of the tangents at the pole and sketch the curve. (4)

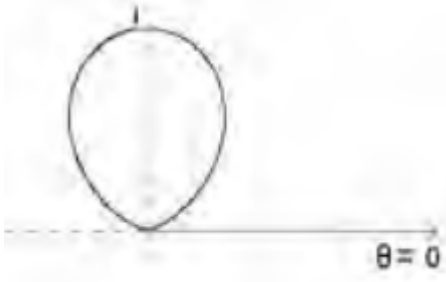
2. $r = \sqrt{3} + \tan \theta$, for $-\frac{1}{3}\pi \leq \theta \leq \frac{1}{4}\pi$. Find the equation of the tangent at the pole. (2)

3. $r = 1 + \cos 2\theta$, for $0 \leq \theta \leq 2\pi$. Find the equations of the tangents at the pole. (2)



Solutions

1.

Solve for $r=0$ for at least one θ θ need not be correct	M1
Get $(\theta) = 0$ and π Ignore extra answers out of range	A1
 <p style="margin-left: 100px;">General shape (symmetry stated or approximately seen)</p>	B1
Tangents at $\theta=0, \pi$ and max r seen	B1

2.

Attempt to solve $r=0, \tan \theta = -\sqrt{3}$ Allow $\pm\sqrt{3}$	M1
Get $\theta = -\frac{1}{3}\pi$ only Allow -60°	A1

3.

Solve $r = 0$ for θ , giving $\theta = \frac{1}{2}\pi$ and $\frac{3}{2}\pi$ Two θ needed (rads only); ignore θ out of range	M1A1
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