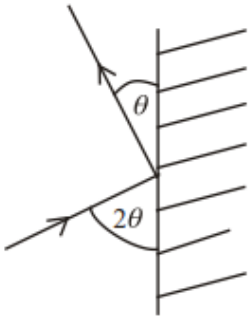
Further Maths A-Level Starter Activity

Topic: Oblique Impact with a Fixed Surface (1)

Chapter Reference: Further Mechanics 1, Chapter 5

10 minutes

1. A small smooth ball B, moving on a horizontal plane, collides with a fixed vertical wall. Immediately before the collision the angle between the direction of motion of B and the wall is 2θ where $0° < \theta < 45°$. Immediately after the collision the angle between the direction of motion of B and the wall is θ as shown in the diagram below. Given that the coefficient of restitution between B and the wall is $\frac{3}{8}$, find the value of $\tan\theta$. **(8)**



Solutions

1.

$u\cos 2\theta = v\cos\theta$	**M1A1**
$\frac{3}{8}u\sin 2\theta = v\sin\theta$	**M1A1**
$3\tan 2\theta = 8\tan\theta$	**M1**
$\frac{6\tan\theta}{1-\tan^2\theta} = 8\tan\theta$	**M1**
$\tan^2\theta = \frac{1}{4}$ $(\tan\theta \neq 0)$ $\tan\theta = \frac{1}{2}$	**M1A1**

Further Maths A-Level Starter Activity

Topic: Oblique Impact with a Fixed Surface (2)

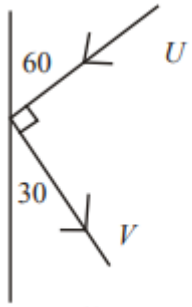
Chapter Reference: Further Mechanics 1, Chapter 5

10 minutes

1. A small ball is moving on a horizontal plane when it strikes a smooth vertical wall. The coefficient of restitution between the ball and the wall is e. Immediately before the impact the direction of motion of the ball makes an angle of 60° with the wall. Immediately after the impact the direction of motion of the ball makes an angle of 30° with the wall.
   a. Find the fraction of the kinetic energy of the ball which is lost in the impact. **(6)**
   b. Find the value of e. **(4)**

Solutions

1a.

 $u\cos 60° = v\cos 30°$	**M1A1**
$u = v\sqrt{3}$ u in terms of v or v.v. – not necessarily simplified. *or ration of the two variables correct*	**A1**
KE lost $= \frac{1}{2}m(u^2 - v^2)$ expression for KE lost	**M1**
Fraction of KE lost $= 1-\left(\frac{v}{u}\right)^2$ expression in one variable for fraction of KE lost – could be u/v as above	**DM1**
$= 1-\frac{1}{3} = \frac{2}{3}$ or at least 3sf ending in 7 or $\frac{3}{4}(1-e^2)$ cao	**A1**

The first three marks can be awarded in (b) if not seen in (a)

1b.

$e = \frac{v\sin 30°}{u\sin 60°}$ Use NIL perpendicular to the wall and form equation in e Correct unsimplified expression as above or $eu\sin 60° = v\sin 30°$ or equivalent	**M1A1**
$= \frac{v}{u}\cdot\frac{1}{\sqrt{3}}$ Substitute values for trig functions or use relationship from (a) and rearrange to e = ...	**DM1**
$= \frac{1}{3}$ cao accept decimals to at least 3sf	**A1**

The first two marks can be awarded in (a)

Further Maths A-Level Starter Activity

Topic: Successive Oblique Impacts (3)

Chapter Reference: Further Mechanics 1, Chapter 5

10 minutes

1. Two smooth vertical walls stand on a smooth horizontal surface and intersect at right angles. A smooth sphere of mass 0.05 kg is moving in the *xy*-plane such that it collides with the first wall at a speed of 0.3 m s^{-1} at an angle of 30° to the wall. The coefficient of restitution between the sphere and both walls is e. Given that after the first collision the sphere is moving with speed 0.27 m s^{-1}, work out:

a. The direction in which the sphere is moving to 3 s.f. **(2)**

b. The value of e. **(2)**

c. The sphere then moves on to collide with the second wall. Calculate the kinetic energy of the sphere after the second collision. **(6)**

Solutions

1a.

First collision – let the angle the spheres make with the wall after the collision be α. For motion parallel to the wall: $0.27\cos\alpha = 0.3\cos 30°$	**M1**
$\alpha = 15.8$ (3 s.f.) After the impact with the first wall, the sphere moves at 15.8° to the wall.	**A1**

1b.

For motion perpendicular to the wall $0.27\sin\alpha = 0.3e\sin 30°$ Dividing this equation by the first gives $\tan\alpha = e\tan 30°$	**M1**
$e = \frac{\tan\alpha}{\tan 30°} = \frac{\tan 15.79°}{\tan 30°} = 0.490$ (3 s.f.)	**A1**

1c.

First collision – let the angle the sphere makes with the wall after the collision be β and its speed after the collision be v For motion parallel to the wall $v\cos\beta = 0.27\cos(90° - \alpha) = 0.23\sin\alpha$	**M1**
For motion perpendicular to the wall $v\sin\beta = 0.27e\sin(90° - \alpha) = 0.27e\cos\alpha$	**M1**
Squaring and adding the two previous equations gives $v^2 cos^2\beta + v^2\sin^2\beta = 0.27^2\sin^2\alpha + 0.27^2e^2\cos^2\alpha$	**M1**
$v^2(\cos^2\beta + \sin^2\beta) = 0.27^2(1 - \cos^2\alpha + e^2\cos^2\alpha)$	**M1**
$v^2 = 0.0729(1 - 0.96225^2 + 0.48990^2 \times 0.96225^2) = 0.0216$ ...	**A1**
Kinetic energy after second collision $= \frac{1}{2}mv^2 = \frac{1}{2} \times 0.05 \times 0.0216 = 0.0054$	**A1**

Further Maths A-Level Starter Activity

Topic: Oblique Impacts of Smooth Spheres (4)

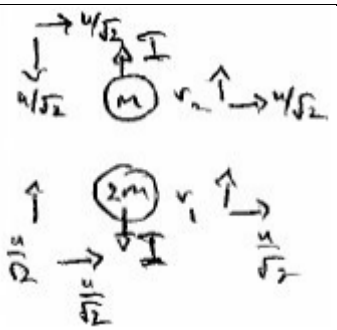
Chapter Reference: Further Mechanics 1, Chapter 5

10 minutes

1. Two small smooth spheres *A* and *B*, of equal size and of mass *m* and 2*m* respectively, are moving initially with the same speed *U* on a smooth horizontal floor. The spheres collide when their centres are on a line *L*. Before the collision the spheres are moving toward each other, with their directions of motion perpendicular to each other and each inclined at an angle of 45° to the line *L*, as shown in Figure 1. The coefficient of restitution between the spheres is $\frac{1}{2}$. Find the magnitude of the impulse which acts on *A* in the collision. **(9)**

Solutions

1.

 Form: $I = m(v_2 + \frac{u}{\sqrt{2}})$	**M1A1**
$CLM(\uparrow): \frac{2mu}{\sqrt{2}} - \frac{mu}{\sqrt{2}} = 2mv_1 + mv_2$ $\frac{u}{\sqrt{2}} = 2v_1 + v_2$ ①	**M1A1**
$NIL: e\frac{2u}{\sqrt{2}} = \frac{u}{\sqrt{2}} = -v_1 + v_2$ ②	**M1A1**
$\Rightarrow \frac{\cancel{2}u}{\sqrt{2}} = \cancel{3}v_2$	**M1A1**
$= mu\sqrt{2}$	**A1**

Further Maths A-Level Starter Activity

Topic: Oblique Impacts of Smooth Spheres (5)
Chapter Reference: Further Mechanics 1, Chapter 5

10 minutes

1. Two smooth uniforms spheres S and T have equal radii. The mass of S is 0.3 kg and the mass of T is 0.6 kg. The spheres are moving on a smooth horizontal plane and collide obliquely. Immediately before the collision the velocity of S is $\mathbf{u}_1$ m s^{-1} and the velocity of T is $\mathbf{u}_2$ m s^{-1}. The coefficient of restitution between the spheres is 0.5 Immediately after the collision the velocity of S is $(-\mathbf{i} + 2\mathbf{j})$ m s^{-1} and the velocity of T is $(\mathbf{i} + \mathbf{j})$ m s^{-1}. Given that when the spheres collide the line joining their centres is parallel to $\mathbf{i}$,

a. Find

i. u_1

ii. u_2 **(6)**

b. After the collision, T goes on to collide with a smooth vertical wall which is parallel to $\mathbf{j}$. Given that the coefficient of restitution between T and the wall is also 0.5, find the angle through which the direction of motion of T is deflected as a result of the collision with the wall. **(5)**

Solutions

1a.

$\uparrow 2$ $\quad$ $\uparrow 1$ $1 \leftarrow$ $\quad$ $\rightarrow 1$ S 0.3 kg $\quad$ T 0.6 kg $2 \uparrow$ $\quad$ $\uparrow 1$ $\rightarrow v \quad w \leftarrow$ $0.3v - 0.6w = 0.3$	**M1A1**
$v - 2w = 1$ $\frac{1}{2}(v + w) = 2$	**M1A1**
$v + w = 4$ $w = 1, v = 3$ (i) $\mathbf{u}_1 = 3\mathbf{i} + 2\mathbf{j}$ (ii) $\mathbf{u}_2 = -\mathbf{i} + \mathbf{j}$	**M1A1**

1b.

$\uparrow 1$ $v \leftarrow$ $\quad$ $v = 0.5$ $1 \uparrow$ $\rightarrow 1$	**B1**
 $\tan\theta = 0.5$ $\quad$ $\tan\theta =$ their v	**M1**
$\theta = 26.6$	**A1**
their $\theta + 45°$	**M1**
Defln angle $= 45 + 26.6 = 71.6°$	**A1**