

Solutions

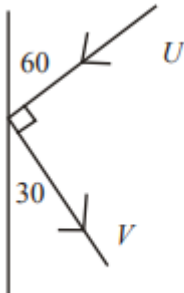
1.

$u \cos 2\theta = v \cos \theta$	M1A1
$\frac{3}{8} u \sin 2\theta = v \sin \theta$	M1A1
$3 \tan 2\theta = 8 \tan \theta$	M1
$\frac{6 \tan \theta}{1 - \tan^2 \theta} = 8 \tan \theta$	M1
$\tan^2 \theta = \frac{1}{4} \quad (\tan \theta \neq 0)$ $\tan \theta = \frac{1}{2}$	M1A1



Solutions

1a.

 <p>$u \cos 60^\circ = v \cos 30^\circ$</p>	M1A1
$u = v\sqrt{3}$ <i>u in terms of v or v.v. – not necessarily simplified. or ratio of the two variables correct</i>	A1
$KE \text{ lost} = \frac{1}{2}m(u^2 - v^2)$ <i>expression for KE lost</i>	M1
$\text{Fraction of KE lost} = 1 - \left(\frac{v}{u}\right)^2$ <i>expression in one variable for fraction of KE lost – could be u/v as above</i>	DM1
$= 1 - \frac{1}{3} = \frac{2}{3}$ <i>or at least 3sf ending in 7 or $\frac{3}{4}(1 - e^2)$ cao</i>	A1

The first three marks can be awarded in (b) if not seen in (a)

1b.

$e = \frac{v \sin 30^\circ}{u \sin 60^\circ}$ <i>Use NIL perpendicular to the wall and form equation in e Correct unsimplified expression as above or $eu \sin 60^\circ = v \sin 30^\circ$ or equivalent</i>	M1A1
$= \frac{v}{u} \cdot \frac{1}{\sqrt{3}}$ <i>Substitute values for trig functions or use relationship from (a) and rearrange to e =</i>	DM1
$= \frac{1}{3}$ <i>cao accept decimals to at least 3sf</i>	A1

The first two marks can be awarded in (a)

Further Maths
A-Level Starter
Activity

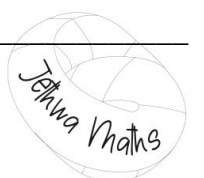


Topic: Successive Oblique Impacts (3)
Chapter Reference: Further Mechanics 1, Chapter 5

10
minutes

1. Two smooth vertical walls stand on a smooth horizontal surface and intersect at right angles. A smooth sphere of mass 0.05 kg is moving in the xy -plane such that it collides with the first wall at a speed of 0.3 m s^{-1} at an angle of 30° to the wall. The coefficient of restitution between the sphere and both walls is e . Given that after the first collision the sphere is moving with speed 0.27 m s^{-1} , work out:

- a. The direction in which the sphere is moving to 3 s.f. **(2)**
- b. The value of e . **(2)**
- c. The sphere then moves on to collide with the second wall. Calculate the kinetic energy of the sphere after the second collision. **(6)**



Solutions

1a.

First collision – let the angle the spheres make with the wall after the collision be α . For motion parallel to the wall: $0.27 \cos \alpha = 0.3 \cos 30^\circ$	M1
$\alpha = 15.8$ (3 s.f.) After the impact with the first wall, the sphere moves at 15.8° to the wall.	A1

1b.

For motion perpendicular to the wall $0.27 \sin \alpha = 0.3e \sin 30^\circ$ Dividing this equation by the first gives $\tan \alpha = e \tan 30^\circ$	M1
$e = \frac{\tan \alpha}{\tan 30^\circ} = \frac{\tan 15.79^\circ}{\tan 30^\circ} = 0.490$ (3 s.f.)	A1

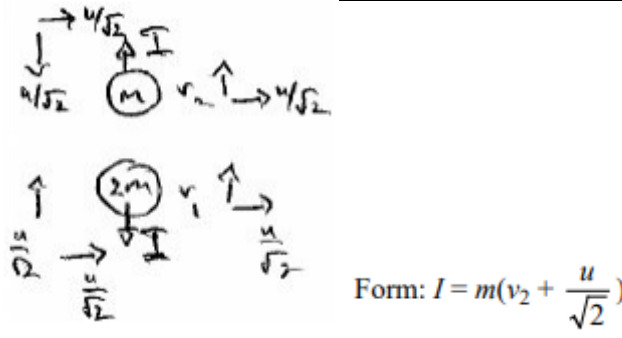
1c.

First collision – let the angle the sphere makes with the wall after the collision be β and its speed after the collision be v For motion parallel to the wall $v \cos \beta = 0.27 \cos(90^\circ - \alpha) = 0.27 \sin \alpha$	M1
For motion perpendicular to the wall $v \sin \beta = 0.27e \sin(90^\circ - \alpha) = 0.27e \cos \alpha$	M1
Squaring and adding the two previous equations gives $v^2 \cos^2 \beta + v^2 \sin^2 \beta = 0.27^2 \sin^2 \alpha + 0.27^2 e^2 \cos^2 \alpha$	M1
$v^2 (\cos^2 \beta + \sin^2 \beta) = 0.27^2 (1 - \cos^2 \alpha + e^2 \cos^2 \alpha)$	M1
$v^2 = 0.0729(1 - 0.96225^2 + 0.48990^2 \times 0.96225^2) = 0.0216 \dots$	A1
Kinetic energy after second collision $= \frac{1}{2}mv^2 = \frac{1}{2} \times 0.05 \times 0.0216 = 0.0054$	A1



Solutions

1.

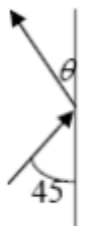
 <p style="margin-left: 200px;">Form: $I = m(v_2 + \frac{u}{\sqrt{2}})$</p>	M1A1
$CLM(\uparrow): \frac{2mu}{\sqrt{2}} - \frac{mu}{\sqrt{2}} = 2mv_1 + mv_2$	M1A1
$\frac{u}{\sqrt{2}} = 2v_1 + v_2 \quad \textcircled{1}$	M1A1
$NIL: e \frac{2u}{\sqrt{2}} = \frac{u}{\sqrt{2}} = -v_1 + v_2 \quad \textcircled{2}$	M1A1
$\Rightarrow \frac{\cancel{2}u}{\sqrt{2}} = \cancel{2}v_2$	M1A1
$= mu\sqrt{2}$	A1

Solutions

1a.

$ \begin{array}{ccc} \uparrow 2 & & \uparrow 1 \\ 1 \leftarrow & & \rightarrow 1 \\ S \ 0.3 \text{ kg} & & T \ 0.6 \text{ kg} \\ 2 \uparrow & & \uparrow 1 \\ & \rightarrow v & w \leftarrow \end{array} $ $0.3v - 0.6w = 0.3$	M1A1
$v - 2w = 1$ $\frac{1}{2}(v + w) = 2$	M1A1
$v + w = 4$ $w = 1, v = 3$ <p>(i) $\mathbf{u}_1 = 3\mathbf{i} + 2\mathbf{j}$ (ii) $\mathbf{u}_2 = -\mathbf{i} + \mathbf{j}$</p>	M1A1

1b.

$ \begin{array}{ccc} & \uparrow 1 & \\ v \leftarrow & & \\ & 1 \uparrow & \\ & & \rightarrow 1 \end{array} $ $v = 0.5$	B1
 $\tan \theta = 0.5$ $\tan \theta = \text{their } v$	M1
$\theta = 26.6$	A1
their $\theta + 45^\circ$	M1
Defln angle = $45 + 26.6 = 71.6^\circ$	A1

