Further Maths A-Level Starter Activity

Topic: Roots of Polynomials (1)

Chapter Reference: Core Pure 1, Chapter 4

6 minutes

1. The quadratic equation

$$x^2 + px + q = 0$$

in which the coefficients p and q are real, has a complex root $\sqrt{5} - i$.

a. Write down the other root of the equation **(1)**

b. Find the sum and product of the two roots of the equation. **(3)**

c. Hence state the values of p and q. **(2)**

Solutions

1a.

$\sqrt{5} + i$	**B1**

1b.

Sum of roots is $2\sqrt{5}$ Product is 6	**B1** **M1A1**

1c.

$p = -2\sqrt{5}$, $q = 6$	**B1** **B1**

Further Maths A-Level Starter Activity

Topic: Roots of Polynomials (2)

Chapter Reference: Core Pure 1, Chapter 4

10 minutes

1. The cubic equation $3x^3-9x^2+6x+2=0$ has roots α, β and γ.
Write down the values of α + β + γ, αβ + βγ + γα and αβγ. **(3)**

2. The cubic equation $3z^3 + pz^2 + 17z + q = 0$, where p and q are real, has a root $\alpha = 1 + 2i$.

a. Write down the value of another non-real root, β, of this equation. **(1)**

b. Find the value of the third root, γ, of this equation. **(3)**

c. Find the values of p and q. **(3)**

Solutions

1.

$\alpha + \beta + \gamma = 3$ $\alpha\beta + \beta\gamma + \gamma\alpha = 2$ $\alpha\beta\gamma = -\frac{2}{3}$	**B1** **B1** **B1**

2a.

$1 - 2i$	**B1**

2b.

$\sum \alpha\beta = \frac{17}{3}$ $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{17}{3}$ $\Rightarrow \gamma = \frac{1}{3}$	**B1** **M1** **A1**

2c.

$\alpha + \beta + \gamma = \frac{-p}{3}$, $\alpha\beta\gamma = \frac{-q}{3}$ $p = -7$ $q = -5$	**M1** **A1** **A1**

Further Maths A-Level Starter Activity

Topic: Roots of Polynomials (3)

Chapter Reference: Core Pure 1, Chapter 4

9 minutes

1. The equation,

$$x^4 - 6x^3 - 73x^2 + kx + m = 0$$

has two positive roots α, β and two negative roots γ, δ. It is given that $\alpha\beta = \gamma\delta = 4$.

a. Find the values of the constant k and m. **(5)**

b. Show that $(\alpha + \beta)(\gamma + \delta) = -81$ **(4)**

Solutions

1a.

$\alpha + \beta + \gamma + \delta = 6$	**B1**
$k = -(\beta\gamma\delta + \alpha\gamma\delta + \alpha\beta\delta + \alpha\beta\gamma)$	**M1**
$= -4(\beta + \alpha + \delta + \gamma)$	**M1**
$= -24$	**A1**
$m = \alpha\beta\gamma\delta = 16$	**B1**

1b.

$\sum\alpha\beta = -73$ $(\alpha + \beta)(\gamma + \delta) = \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta$ $\sum\alpha\beta - \alpha\beta - \gamma\delta$ $= -73 - 4 - 4$ $= -81$	**B1** **M1** **A1** **A1**

Further Maths A-Level Starter Activity

Topic: Roots of Polynomials (4) Chapter Reference: Core Pure 1, Chapter 4	8 minutes

1. Show that $(\alpha\beta + \beta\gamma + \gamma\alpha)^2 \equiv \alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 + 2\alpha\beta\gamma(\alpha + \beta + \gamma)$. **(3)**

2. It is given that α, β and γ are the roots of the cubic equation $x^3 + px^2 - 4x + 3 = 0$, where p is a constant.

   Find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$ in terms of p. **(5)**

Solutions

1.

Attempt at complete expansion	**M1**
Obtain correct unsimplified answer	**A1**
Obtain **given** answer correctly	**A1**

2.

$\sum\alpha = -p, \Sigma\alpha\beta = -4, \alpha\beta\gamma\ = -3$	**B1**
	M1
	A1
$\dfrac{16\ -\ 6p}{9}$	**M1**
	A1

Further Maths A-Level Starter Activity

Topic: Roots of Polynomials (5) Chapter Reference: Core Pure 1, Chapter 4	8 minutes

1. The cubic equation $3x^3 - 9x^2 + 6x + 2 = 0$ has root α, β and γ.

a. Write down the values of $\alpha + \beta + \gamma$ and $\alpha\beta + \beta\gamma + \gamma\alpha$. **(2)**

b. Find the value of $\alpha^2 + \beta^2 + \gamma^2$. **(2)**

c. Use the substitution $x = \frac{1}{u}$ to find a cubic equation in u with integer coefficients. **(2)**

d. Use your answer to part (c) to find the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$. **(2)**

Solutions

1a.

$\alpha + \beta + \gamma = 3, \alpha\beta + \beta\gamma + \gamma\alpha = 2$	**B1** **B1**

1b.

$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$ $= 9 - 4 = 5$	**M1** **A1**

1c.

$\frac{3}{u^3} - \frac{9}{u^2} + \frac{6}{u} + 2 = 0$ $2u^3 + 6u^2 - 9u + 3 = 0$	**M1** **M1**

1d.

$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = -3$	**M1** **A1**