Further Maths A-Level Starter Activity

Topic: Direct Impact and Newton's Law of Restitution (1)

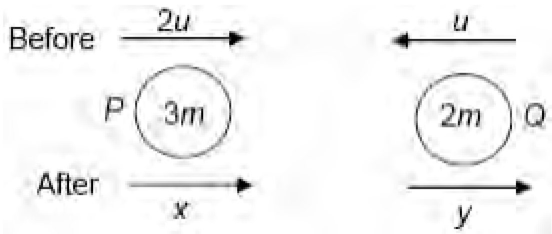
Chapter Reference: Further Mechanics 1, Chapter 4

10 minutes

1. A particle P of mass $3m$ is moving in a straight line withs speed $2u$ on a smooth horizontal table. It collides directly with another particle Q of mass $2m$ which is moving with speed u in the opposite direction to P. The coefficient of restitution between P and Q is e.
   a. Show that the speed of Q immediately after the collision is $\frac{1}{5}(9e + 4)u$. **(5)**
   b. The speed of P immediately after the collision is $\frac{1}{2}u$. Show that $e = \frac{1}{4}$. **(4)**

Solutions

1a.

 Correct use of NEL	**M1**
$y - x = e(2u + u)$ o.e.	**A1**
CLM (→): $3m(2u) + 2m(-u) = 3m(x) + 2m(y)$ ($\Rightarrow 4u = 3x + 2y$)	**B1**
Hence $x = y - 3eu$, $4u = 3(y - 3eu) + 2y$, ($u(9e + 4) = 5y$)	**M1**
Hence, speed of Q = $\underline{\frac{1}{5}(9e+4)u}$ **AG**	**A1**

1b. Either

$x = y - 3eu = \frac{1}{5}(9e+4)u - 3eu$	**M1**
Hence, speed $\text{P} = \frac{1}{5}(4-6e)u = \frac{2u}{5}(2-3e)$ o.e.	**A1**
$x = \frac{1}{2}u = \frac{2u}{5}(2-3e) \Rightarrow 5u = 8u - 12eu, \Rightarrow 12e = 3$ & solve for e	**M1**
gives, $e = \frac{3}{12} \Rightarrow \underline{\mathbf{e = \frac{1}{4}}}$ **AG**	**A1**

Or

Using NEL correctly with given speeds of P and Q	**M1**
$3eu = \frac{1}{5}(9e+4)u - \frac{1}{2}u$	**A1**
$3eu = \frac{9}{5}eu + \frac{4}{5}u - \frac{1}{2}u$, $3e - \frac{9}{5}e = \frac{4}{5} - \frac{1}{2}$ & solve for e	**M1**
$\frac{6}{5}e = \frac{3}{10} \Rightarrow e = \frac{15}{60} \Rightarrow e = \frac{1}{4}$.	**A1**

Further Maths A-Level Starter Activity

Topic: Direct Collision with a Smooth Plane (2)

Chapter Reference: Further Mechanics 1, Chapter 4

10 minutes

1. A small sphere of mass 0.2kg is projected vertically downwards with speed 21 m s^{-1} from a point at a height of 40 m above horizontal ground. It hits the ground and rebounds vertically upwards, coming to instantaneous rest at its initial point of projection. Ignoring air resistance, calculate
    a. the coefficient of restitution between the sphere and the ground, **(6)**
    b. the magnitude of the impulse which the ground exerts on the sphere. **(2)**

Solutions

1a.

$x^2 = 21^2 + 2x40x9.8$	**M1**
$x = 35$	**A1**
$0 = y^2 – 2x40x9.8$	**M1**
$y = 28$ may be implied	**A1**
e = 28/35	**M1**
e = 0.8 aef	**A1**

1b.

0.2x28 – – 0.2x35 must be double negative	**M1**
I = 12.6	**A1**

Topic: Loss of Kinetic Energy (3)

Chapter Reference: Further Mechanics 1, Chapter 4

10 minutes

1. Two spheres of the same radius with masses 2 kg and 3 kg are moving directly towards each other on a smooth horizontal plane with speeds 8 m s^{-1} and 4 m s^{-1} respectively. The spheres collide and the kinetic energy lost is 81 J. Calculate the speed and direction of motion of each sphere after the collision. **(12)**

Solutions

1.

$16 - 12 = 2x + 3y$	**M1**
$4 = 2x + 3y$ aef	**A1**
$½.2(8)^2 + ½.3(4)^2$ or $½.2x^2 + ½..3y^2$ or $±½.2(8^2 - x^2)$ or $±½.3(4^2 - y^2)$	**B1**
$½.2(8)^2 + ½.3(4)^2 - ½.2x^2 - ½...3y^2 = 81$	**M1**
$2x^2 + 3y^2 = 14$ aef	**A1**
Attempt to eliminate x or y from a linear and a quadratic equation	**M1**
$15y^2 - 24y - 12 = 0$ or $10x^2 - 16x - 26 = 0$ aef	**A1**
Attempt to solve a three term quadratic	**M1**
$x = -1$ (or $x = 2.6$)	**A1**
$y = 2$ (or $y = -2/5$)	**A1**
$x = -1$ and $y = 2$ only	**A1**
speeds 1, 2 away from each other	**A1**

Topic: Successive Direct Impacts (4)

Chapter Reference: Further Mechanics 1, Chapter 4

10 minutes

1. A small ball of mass 0.5 kg is held at a height of 3.136 m above a horizontal floor. The ball is released from rest and rebounds from the floor. The coefficient of restitution between the ball and floor is e. The speed of the ball in terms of e immediately after the impact is $7.84e$. The ball continues to bounce until it eventually comes to rest.
   a. Show that the time between the first bounce and the second bounce is $1.6e$. **(2)**
   b. Write down, in terms of e, the time between
      i. the second bounce and the third bounce,
      ii. the third bounces and the fourth bounce. **(2)**
   c. Given that the time from the ball being released until it comes to rest is 5 s, find the value of e. **(5)**

Solutions

1a.

$-7.84e = 7.84e - gt$ Uses a complete method to find t.	**M1**
$t = 1.6e$	**A2**

1b.

(a)	$t_2 = 1.6e^2$	**B1**
(b)	$t_3 = 1.6e^3$	**B1**

1c.

Time to first bounce is 0.8 s	**B1**
Identify total time is sum of a GP in e Indication of the sum of at least to term in e^4	**B1**
Equate 3.4 or 4.2 or 5 or 5.8 with attempt at use of formula for sum to infinity of a GP.	**M1**
$\frac{1.6e}{1-e} = 4.2$	**A1**
$e = 0.724$ Allow 21/29	**A1**

Further Maths A-Level Starter Activity

Topic: Successive Direct Impacts (5) Chapter Reference: Further Mechanics 1, Chapter 4	10 minutes

1. Three smooth spheres A, B and C, of equal radius and of masses m kg, $2m$ kg and $3m$ kg respectively, lie in a straight line and are free to move on a smooth horizontal table. Sphere A is moving with speed 5 m s^{-1} when it collides directly with sphere B which is stationary. As a result of the collision B starts to move with speed 2ms^{-1}.
   a. Find the coefficient of restitution between A and B. **(4)**
   b. Find, in terms of m, the magnitude of the impulse that A exerts on B and state the direction of this impulse. **(2)**
   c. Sphere B subsequently collides with sphere C which is stationary. As a result of this impact B and C coalesce. Show that there will be another collision. **(3)**

<u>Solutions</u>

1a.

5m = mu + 4m cons. of mom.	**M1**
u = 1	**A1**
e = (2-1)/5	**M1**
$e = \frac{1}{5}$	**A1**

1b.

I = 4m	**B1**
$\rightarrow$ to the right	**B1**

1c.

4m = 5mv	**M1**
$v = \frac{4}{5}$	**A1**
$\frac{4}{5} < 1$	**B1**