Further Maths A-Level Starter Activity

Topic: Sums of Squares and Cubes (1)

Chapter Reference: Core Pure 1, Chapter 3

10 minutes

1. Find $\sum_{r=1}^{n} r(r+1)(r-2)$, expressing your answer in a fully factorised form. **(6)**

2. Find $\sum_{r=1}^{n} (2r-1)^2$, expressing your answer in a fully factorised form. **(6)**

Solutions

1.

Express as sum of three series	**M1**
Use standard results	**M1**
$\frac{1}{4}n^2(n+1)^2-\frac{1}{6}n(n+1)(2n+1)-n(n+1)$	**A1**
Attempt to factorise	**M1**
Obtain at least factor of n(n+1)	**A1**
$\frac{1}{12}n(n+1)(n+2)(3n-7)$	**A1**

2. Either

Express as a sum of 3 terms	**M1**
Use standard sum results	**M1**
$\frac{2}{3}n(n+1)(2n+1)-2n(n+1)+n$	**A1**
Attempt to factorise	**M1**
Obtain at least factor of n and a quadratic	**A1**
$\frac{1}{3}n(2n-1)(2n+1)$	**A1**

Or

$\sum_{r=1}^{2n} r^2 - 4\sum_{r=1}^{n} r^2$	**M1**
Use standard result	**M1**
$\frac{1}{6}\times 2n(2n+1)(4n+1)-4\times\frac{1}{6}n(n+1)(2n+1)$	**A1**
Attempt to factorise	**M1**
Obtain at least factor of n	**A1**
$\frac{1}{3}n(2n-1)(2n+1)$	**A1**

Topic: Sums of Squares and Cubes (2)

Chapter Reference: Core Pure 1, Chapter 3

10 minutes

1. (a) (i) Expand $(2r-1)^2$. **(1)**

(ii) Hence show that $\sum_{r=1}^{n}(2r-1)^2 = \frac{1}{3}n(4n^2-1)$ **(5)**

(b) Hence find the sum of the squares of the odd numbers between 100 and 200. **(4)**

Solutions

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$(2r-1)^2 = 4r^2 - 4r + 1$	**B1**

1aii.

$\sum(2r-1)^2 = 4\sum r^2 - 4\sum r + \sum 1$	**M1**
$\ldots = \frac{4}{3}n^3 - \frac{4}{3}n + \sum 1$	**M1** **A1**
$\sum 1 = n$	**B1**
Result convincingly shown	**A1**

1b.

$\text{Sum} = f(100) - f(50)$	**M1** **A1**
$\ldots = 1166\,650$	**A2**

Topic: Sums of Squares and Cubes (3)

Chapter Reference: Core Pure 1, Chapter 3

10 minutes

1. Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that, for all positive integers n, **(6)**

$$\sum_{r=1}^{n}(6r^2 + 2r + 1) = n(2n^2 + 4n + 3).$$

2. Show that $\sum_{r=1}^{n}(r^2 - r) = kn(n+1)(n-1)$ where k is a rational number. **(4)**

Solutions

1.

$\Sigma(r^2 - r) = \Sigma r^2 - \Sigma r$	**M1**
At least one linear factor found	**M1**
$\Sigma(r^2 - r) = \frac{1}{6}n(n+1)(2n+1-3)$	**M1**
$\ldots = \frac{1}{3}n(n+1)(n-1)$	**A1**

2.

$6\Sigma r^2 + 2\Sigma r + \Sigma 1$	**M1**
$6\Sigma r^2 = n(n+1)(2n+1)$	**A1**
$2\Sigma r = n(n+1)$	**A1**
$\Sigma 1 = n$	**A1**
$n(2n^2 + 4n + 3)$	**M1**
Obtain given answer correctly	**A1**

Topic: Sums of Squares and Cubes (4)

Chapter Reference: Core Pure 1, Chapter 3

11 minutes

1. Use the standard results for $\sum_{r=1}^{n} r$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$ to show that, for all positive integers n, **(6)**

$$\sum_{r=1}^{n}(8r^3 - 6r^2 + 2r) = 2n^3(n+1).$$

2. Use the standard results for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r^2$ to show that, for all positive integers n, **(5)**

$$\sum_{r=1}^{n}(r^3 + r^2) = \frac{1}{12}n(n+1)(n+2)(3n+1).$$

Solutions

1.

$8\Sigma r^3 - 6\Sigma r^2 + 2\Sigma r$	**M1**
$8\Sigma r^3 = 2n^2(n+1)^2$	**A1**
$6\Sigma r^2 = n(n+1)(2n+1)$	**A1**
$2\Sigma r = n(n+1)$	**A1**
$2n^3(n+1)$	**M1**
Obtain given answer correctly	**A1**

2.

$\Sigma r^3 + \Sigma r^2$	**M1**
$\Sigma r^2 = \frac{1}{6}n(n+1)(2n+1)$	**A1**
$\Sigma r^3 = \frac{1}{4}n^2(n+1)^2$	**A1**
$\frac{1}{12}n(n+1)(n+2)(3n+1)$	**M1**
Obtain given answer correctly or complete verification	**A1**

Further Maths A-Level Starter Activity

Topic: Sums of Squares and Cubes (5)

Chapter Reference: Core Pure 1, Chapter 3

10 minutes

1. Find $\sum_{r=1}^{n}(3r^2 - 3r + 2)$, expressing your answer in a fully factorised form. **(7)**

2. Evaluate $\sum_{r=101}^{250} r^3$. **(3)**

Solutions

1.

Express as sum of 3 series	**M1**
Use standard series results, at least 1 correct	**M1**
Two terms correct	**A1**
$\frac{1}{2}n(n+1)(2n+1) - \frac{3}{2}n(n+1) + 2n$	**A1**
Obtain factor of *n*	**M1**
$n(n^2+1)$ Allow A1 for $\frac{1}{2(2n^2+2)}$	**A2**

2.

State correct value of S_{250} or S_{100}	**B1**
Subtract $S_{250} - S_{100}$ (or S_{101} or S_{99})	**M1**
984390625 – 25502500 = 958888125	**A1**