

**Further Maths
A-Level Starter
Activity**



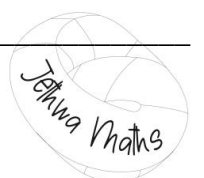
Topic: Sums of Squares and Cubes (1)

Chapter Reference: Core Pure 1, Chapter 3

**10
minutes**

1. Find $\sum_{r=1}^n r(r+1)(r-2)$, expressing your answer in a fully factorised form. (6)

2. Find $\sum_{r=1}^n (2r-1)^2$, expressing your answer in a fully factorised form. (6)



Solutions

1.

Express as sum of three series	M1
Use standard results	M1
$\frac{1}{4}n^2(n+1)^2 - \frac{1}{6}n(n+1)(2n+1) - n(n+1)$	A1
Attempt to factorise	M1
Obtain at least factor of $n(n+1)$	A1
$\frac{1}{12}n(n+1)(n+2)(3n-7)$	A1

2. Either

Express as a sum of 3 terms	M1
Use standard sum results	M1
$\frac{2}{3}n(n+1)(2n+1) - 2n(n+1) + n$	A1
Attempt to factorise	M1
Obtain at least factor of n and a quadratic	A1
$\frac{1}{3}n(2n-1)(2n+1)$	A1

Or

$\sum_{r=1}^{2n} r^2 - 4 \sum_{r=1}^n r^2$	M1
Use standard result	M1
$\frac{1}{6} \times 2n(2n+1)(4n+1) - 4 \times \frac{1}{6}n(n+1)(2n+1)$	A1
Attempt to factorise	M1
Obtain at least factor of n	A1
$\frac{1}{3}n(2n-1)(2n+1)$	A1



Solutions

1ai.

$(2r-1)^2 = 4r^2 - 4r + 1$	B1
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$\sum(2r-1)^2 = 4\sum r^2 - 4\sum r + \sum 1$	M1
$\dots = \frac{4}{3}n^3 - \frac{4}{3}n + \sum 1$	M1 A1
$\sum 1 = n$	B1
Result convincingly shown	A1

1b.

$\text{Sum} = f(100) - f(50)$	M1 A1
$\dots = 1\,166\,650$	A2

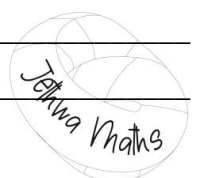




1. Use the standard results for $\sum_{r=1}^n r$ and $\sum_{r=1}^n r^2$ to show that, for all positive integers n , (6)

$$\sum_{r=1}^n (6r^2 + 2r + 1) = n(2n^2 + 4n + 3).$$

2. Show that $\sum_{r=1}^n (r^2 - r) = kn(n + 1)(n - 1)$ where k is a rational number. (4)



Solutions

1.

$\Sigma(r^2 - r) = \Sigma r^2 - \Sigma r$	M1
At least one linear factor found	M1
$\Sigma(r^2 - r) = \frac{1}{6}n(n+1)(2n+1-3)$	M1
$\dots = \frac{1}{3}n(n+1)(n-1)$	A1

2.

$6\Sigma r^2 + 2\Sigma r + \Sigma 1$	M1
$6\Sigma r^2 = n(n+1)(2n+1)$	A1
$2\Sigma r = n(n+1)$	A1
$\Sigma 1 = n$	A1
$n(2n^2 + 4n + 3)$	M1
Obtain given answer correctly	A1



Solutions

1.

$8\Sigma r^3 - 6\Sigma r^2 + 2\Sigma r$	M1
$8\Sigma r^3 = 2n^2(n+1)^2$	A1
$6\Sigma r^2 = n(n+1)(2n+1)$	A1
$2\Sigma r = n(n+1)$	A1
$2n^3(n+1)$	M1
Obtain given answer correctly	A1

2.

$\Sigma r^3 + \Sigma r^2$	M1
$\Sigma r^2 = \frac{1}{6}n(n+1)(2n+1)$	A1
$\Sigma r^3 = \frac{1}{4}n^2(n+1)^2$	A1
$\frac{1}{12}n(n+1)(n+2)(3n+1)$	M1
Obtain given answer correctly or complete verification	A1



Solutions

1.

Express as sum of 3 series	M1
Use standard series results, at least 1 correct	M1
Two terms correct	A1
$\frac{1}{2}n(n+1)(2n+1) - \frac{3}{2}n(n+1) + 2n$	A1
Obtain factor of n	M1
$n(n^2 + 1)$ Allow A1 for $\frac{1}{2(2n^2 + 2)}$	A2

2.

State correct value of S_{250} or S_{100}	B1
Subtract $S_{250} - S_{100}$ (or S_{101} or S_{99})	M1
$984390625 - 25502500 = 958888125$	A1

