Further Maths A-Level Starter Activity

Topic: Methods in Calculus (1)

Chapter Reference: Core Pure 2, Chapter 3

9 minutes

1. Evaluate $\int_1^{\infty} \frac{8}{x^3}\,dx$. **(4)**

2a. Find $\int_1^{\infty} 24x^{-3}\,dx$. **(2)**

b. Given that $\int_a^{\infty} 24x^{-3}\,dx = 3$, find the value of the positive constant a. **(3)**

Solutions

1.

$\int_1^a 8x^{-3}\,dx = \left[\frac{8x^{-2}}{-2}\right]_1^a = [-4x^{-2}]_1^a$ $= -4a^{-2} - -4(1)^{-2} = -4a^{-2} + 4$ As $a \to \infty, a^{-2} \to 0$ $\Rightarrow -4a^{-2} + 4 \to 4$	**M1** **M1** **A1** **A1**

2a.

$\frac{24x^{-2}}{-2} + c = -12x^2 + C$	**M1** **A1**

2b.

$[-12x^{-2}]_a^k = -12k^{-2} + 12a^{-2}$ As $k \to \infty = k^{-2} \to 0$ $\Rightarrow = -12k^{-2} + 12a^{-2} = 12a^{-2}$ $12a^{-2} = 3$ $a^2 = 4$ $\therefore a = 2$ since a is positive	**M1** **M1** **A1**

Further Maths A-Level Starter Activity

Topic: Methods in Calculus (2)

Chapter Reference: Core Pure 2, Chapter 3

8 minutes

1. Find the exact mean value of $f(x) = \frac{x}{(x^2+1)^3}$ over the interval [1,3]. **(4)**

2. A car's speed in feet per second is given by $f(t) = t^{1.7} - 6t + 80$. What's its average speed from $t = 5$ seconds to $t = 15$ seconds? What's that in miles per hour? **(4)**

Solutions

1.

Mean value = $\frac{1}{3-1}\int_1^3 \frac{x}{(x^2+1)^3}\,dx$	**M1**
	M1
Substitution $u = x^2 + 1$	**A1**
$du = 2x\,dx$	
$= \frac{-1}{8}[u^{-2}]_2^{10}$	
= 0.03	**A1**

2.

Mean value = $\frac{1}{15-5}\int_5^{15}(t^{1\cdot7} - 6t + 80)\,dt$	**M1**
	M1
72.62 feet per second	**A1**
≈ 49.51 per hour	**A1**

Further Maths A-Level Starter Activity

Topic: Methods in Calculus (3)

Chapter Reference: Core Pure 2, Chapter 3

6 minutes

1a. Given that $y = \tan^{-1} x$, prove that $\frac{dy}{dx} = \frac{1}{1+x^2}$. **(3)**

1b. Verify that $y = \tan^{-1} x$ satisfies the equation

$$(1 + x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 0.$$

(3)

Solutions

1a.

$\sec^2 y \frac{dy}{dx} = 1$ $1 + \tan^2 y = \sec^2 y$	**M1** **M1** **A1**

1b.

Reasonable attempt to diff. $\frac{-2x}{(1+x^2)^2}$	**M1** **M1** **A1**

Further Maths A-Level Starter Activity

Topic: Methods in Calculus (4)

Chapter Reference: Core Pure 2, Chapter 3

5 minutes

1. By first completing the square in the denominator, find the exact value of

$$\int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{4x^2 - 4x + 5}\, dx.$$

(5)

Solutions

1.

$\int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{(2x-1)^2+4}\,dx$	**B1**
	M1
$= \frac{1}{2}\left[-4\tan^{-1}\frac{2x-1}{2}\right]_{1/2}^{3/2}$	**A1**
$= \frac{1}{4}(\tan^{-1}1 - \tan^{-1}0) = \frac{1}{16}\pi$	**M1** **A1**

Further Maths A-Level Starter Activity

Topic: Methods in Calculus (5)

Chapter Reference: Core Pure 2, Chapter 3

9 minutes

1a. Express $\frac{4}{(1-x)(1+x)(1+x^2)}$ in partial fractions. **(5)**

1b. Show that $\int_0^{\frac{1}{\sqrt{3}}} \frac{4}{(1-x^4)}\,dx = \ln\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right) + \frac{1}{3}\pi$. **(4)**

Solutions

1a.

$4 = A(1+x)(1+x^2) + B(1-x)(1+x^2) + (Cx+D)(1-x)(1+x)$	**M1**
A = 1, B = 1	**M1**
C = 0, D = 2	**M1**
	A1
	A1

1b.

Get $A\ln(1+x) - B\ln(1-x)$	**M1**
Get $D\tan^{-1}x$	**B1**
Use limit in their integrated expressions	**M1**
Clearly get A.G.	**A1**