

Further Maths
A-Level Starter
Activity



Topic: Methods in Calculus (1)

Chapter Reference: Core Pure 2, Chapter 3

**9
minutes**

1. Evaluate $\int_1^{\infty} \frac{8}{x^3} dx$.

(4)

2a. Find $\int_1^{\infty} 24x^{-3} dx$.

(2)

b. Given that $\int_a^{\infty} 24x^{-3} dx = 3$, find the value of the positive constant a .

(3)



Solutions

1.

$\int_1^a 8x^{-3} dx = \left[\frac{8x^{-2}}{-2} \right]_1^a = [-4x^{-2}]_1^a$ $= -4a^{-2} - (-4(1)^{-2}) = -4a^{-2} + 4$ <p>As $a \rightarrow \infty, a^{-2} \rightarrow 0$ $\Rightarrow -4a^{-2} + 4 \rightarrow 4$</p>	M1 M1 A1 A1
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2a.

$\frac{24x^{-2}}{-2} + c = -12x^2 + C$	M1 A1
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2b.

$[-12x^{-2}]_a^k = -12k^{-2} + 12a^{-2}$ <p>As $k \rightarrow \infty = k^{-2} \rightarrow 0$ $\Rightarrow = -12k^{-2} + 12a^{-2} = 12a^{-2}$</p> $12a^{-2} = 3$ $a^2 = 4$ <p>$\therefore a = 2$ since a is positive</p>	M1 M1 A1
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Solutions

1.

Mean value = $\frac{1}{3-1} \int_1^3 \frac{x}{(x^2+1)^3} dx$	M1
Substitution $u = x^2 + 1$	M1
$du = 2x dx$	A1
$= \frac{-1}{8} [u^{-2}]_2^{10}$	
$= 0.03$	A1

2.

Mean value = $\frac{1}{15-5} \int_5^{15} (t^{1.7} - 6t + 80) dt$	M1
72.62 feet per second	M1
≈ 49.51 per hour	A1
	A1



**Further Maths
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Topic: Methods in Calculus (3)

Chapter Reference: Core Pure 2, Chapter 3

**6
minutes**

1a. Given that $y = \tan^{-1} x$, prove that $\frac{dy}{dx} = \frac{1}{1+x^2}$.

(3)

1b. Verify that $y = \tan^{-1} x$ satisfies the equation

$$(1 + x^2) \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} = 0.$$

(3)



Solutions

1a.

$\sec^2 y \frac{dy}{dx} = 1$ $1 + \tan^2 y = \sec^2 y$	M1 M1 A1
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1b.

Reasonable attempt to diff. $\frac{-2x}{(1+x^2)^2}$	M1 M1 A1
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Solutions

1.

$\int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{(2x-1)^2+4} dx$	B1
$= \frac{1}{2} \left[-4 \tan^{-1} \frac{2x-1}{2} \right]_{1/2}^{3/2}$	M1
$= \frac{1}{4} (\tan^{-1} 1 - \tan^{-1} 0) = \frac{1}{16} \pi$	A1
	A1



Solutions

1a.

$4 = A(1+x)(1+x^2) + B(1-x)(1+x^2) + (Cx+D)(1-x)(1+x)$ $A = 1, B = 1$ $C = 0, D = 2$	M1 M1 M1 A1 A1
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1b.

Get $A \ln(1+x) - B \ln(1-x)$ Get $D \tan^{-1} x$ Use limit in their integrated expressions Clearly get A.G.	M1 B1 M1 A1
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