Further Maths A-Level Starter Activity

Topic: Geometric Distribution (1)

Chapter Reference: Further Statistics 1, Chapter 3

10 minutes

1. Andrea practises shots at a goal. For each shot the probability of her scoring a goal is $\frac{2}{5}$. Each shot is independent of other shots.

   Find the probability that she scores her first goal

a. On her 5th shot, **(2)**

b. Before her 5th shot. **(3)**

2. The random variable X has the distribution Geo(0.2). Find

a. $P(X = 3)$, **(2)**

b. $P(3 \leq X \leq 5)$. **(3)**

Solutions

1a.

$(\frac{3}{5})^2 \times \frac{2}{5}$	**M1**
$= 0.0518$ (3sfs) or $\frac{162}{3125}$ (oe)	**A1**

1b.

$(\frac{3}{5})^4$	**M1**
$1 - (\frac{3}{5})^4$	**M1**
$= 0.870$ (3 sfs) or $\frac{544}{625}$ (oe)	**A1**

2a.

$0.8^2 \times 0.2$	**M1**
$= \frac{16}{125}$ or 0.128	**A1**

2b.

$0.8^2 \times 0.2 + 0.8^3 \times 0.2 + 0.8^4 \times 0.2$ 1 term omitted or wrong or extra	**M2** **(M1)**
$= \frac{976}{3125}$ or 0.312 (3 sfs)	**A1**

Further Maths A-Level Starter Activity

Topic: Mean and Variance for Geometric Distribution (2)

Chapter Reference: Further Statistics 1, Chapter 3

8 minutes

1. A random variable T has the distribution Geo($\frac{1}{5}$). Find

a. $P(T > 4)$, **(2)**

b. $E(T)$. **(1)**

2. Danny records X, the number of attempts it takes him to throw a basketball into a hoop.

Given that $P(X = 2) = 0.16$ and that $p < 0.5$, find:

a. p, the probability that Danny dunks the basketball on each single attempt, **(3)**

b. The expected number of attempts Danny tries, **(1)**

c. The variance of X. **(1)**

Solutions

1a.

$(\frac{4}{5})^4$ alone or $1 - (\frac{1}{5} + \frac{4}{5} \times \frac{1}{5} + (\frac{4}{5})^2 \times \frac{1}{5} + (\frac{4}{5})^3 \times \frac{1}{5})$	**M1**
$= \frac{256}{625}$ or 0.410 (3 sfs) (allow 0.41)	**A1**

1b.

$1 \div \frac{1}{5}$ $= 5$	**B1**

2a.

$P(X = 2) = p(1 - p) = 0.16$	**M1**
$p^2 - p + 0.16 = 0$ $p = \frac{1 \pm \sqrt{(-1)^2 - 4(0.16)}}{2} = \frac{1 \pm \sqrt{0.36}}{2} = \frac{1 \pm 0.6}{2}$ $p = 0.2$ or 0.8	**M1**
As $p < 0.5$, solution is $p = 0.2$	**A1**

2b.

$E(X) = \frac{1}{0.2} = 5$	**B1**

2c.

$Var(X) = \frac{1-0.2}{0.2^2} = \frac{0.8}{0.04} = 20$	**B1**

Further Maths A-Level Starter Activity

Topic: Negative Binomial Distribution (3)

Chapter Reference: Further Statistics 1, Chapter 3

6 minutes

1. When Stéphane plays chess against his favourite computer program, he wins with probability 0.60, loses with probability 0.10, and 30% of the games result is a draw. Assume independence.

   Find the probability that Stéphane's fifth win happens when he plays his eighth game. **(2)**

2. Customers come into a store. 30% of them make a purchase.
   Calculate the probability that the second purchase is made by the sixth customer. **(2)**

3. Temi spins a coin, biased towards heads. If the probability of spinning head is 0.65, find the probability that Temi spins her third head on her sixth spin. **(2)**

Solutions

1.

Negative Binomial, $p = 0.60$, $r = 5$ $P(X = 8) = {}^{7}C_{4} \times 0.60^{5} \times 0.40^{3}$	**M1**
$= 0.17418$	**A1**

2.

Negative Binomial, $p = 0.3$, $r = 2$ $P(X = 6) = {}^{5}C_{1} \times 0.3^{2} \times 0.7^{4}$	**M1**
$= 0.108045$	**A1**

3.

Negative Binomial, $p = 0.65$, $r = 3$ $P(X = 6) = {}^{5}C_{2} \times 0.65^{3} \times 0.35^{3}$	**M1**
$= 0.117745$	**A1**

Further Maths A-Level Starter Activity

Topic: Mean and Variance for Negative Binomial Distribution (4)

Chapter Reference: Further Statistics 1, Chapter 3

10 minutes

1. The probability that John wins a coconut in a game at the fair is 0.15.

   John plays a number of games.

a. Find the probability of John winning his second coconut on his 7th game. **(2)**

b. Find the expected number of games John would need to play in order to win 3 coconuts. **(1)**

c. State two assumptions that you made in part a. **(2)**

   Sue plays the same game, but has a different probability of winning a coconut. She plays until she has won r coconuts. The random variable G represents the total number of games Sue plays.

d. Given that the mean and the standard deviation of G are 18 and 6 respectively, determine whether John or Sue has the greater probability of winning a coconut in a game. **(5)**

Solutions

1.

$\binom{6}{1}(0.15)(0.85)^5(0.15)$	**M1**
$= 0.05990$	**A1**

1b.

$\frac{3}{0.15} = 20$	**B1**

1c.

Probability that John wins a coconut in a game is constant Games are independent	**B1** **B1**

1d.

$\frac{r}{p} = 18$	**B1**
$\frac{r(1-p)}{p^2} = 36$	**B1**
$18(1 - p) = 36p$	**M1**
$p = \frac{1}{3} > 0.15$	**A1**
Sue has the greater probability of winning a coconut game	**A1**

Further Maths A-Level Starter Activity

Topic: Geometric and Negative Binomial Distribution (5)

Chapter Reference: Further Statistics 1, Chapter 3

10 minutes

1. Each day Harry makes repeated attempts to light his gas fire. If the fire lights he makes no more attempts. On each attempt, the probability that the fire will light is 0.3 independent of all other attempts. Find the probability that

a. The fire lights on the 5^{th} attempt, **(2)**

b. Harry needs more than 1 attempt but fewer than 5 attempts to light the fire. **(3)**

2. In each round of a game, four fair coins are spun.

a. Find the probability that all four coins show the same result. **(2)**

b. Find the probability that all four coins show the same result for the third time on the sixth trial. **(3)**

Solutions

1a.

$0.7^4 \times 0.3$ alone	**M1**
$= 0.0720$ (3 sf) or $\frac{7203}{100000}$ (oe)	**A1**

1b.

$(0.7 + 0.7^2 + 0.7^3) \times 0.3$ 1 term omitted or wrong or extra	**M2** **(M1)**
$= 0.4599$ or 0.460 (3 sf) or $\frac{4599}{10000}$ (oe)	**A1**

2a.

$(\frac{1}{2})^3$	**M1**
$= \frac{1}{8} = 0.125$	**A1**

2b.

$X\sim$ Negative B(3,0.125)	**M1**
$P(X = 6) = \binom{5}{2} \times (0.125)^3 \times (0.875)^3$	**M1**
$= 0.0131$ (4 d.p.)	**A1**