A-Level Starter Activity

Jethwa Maths

Topic: Completing the Square

Chapter Reference: Pure 1, Chapter 2

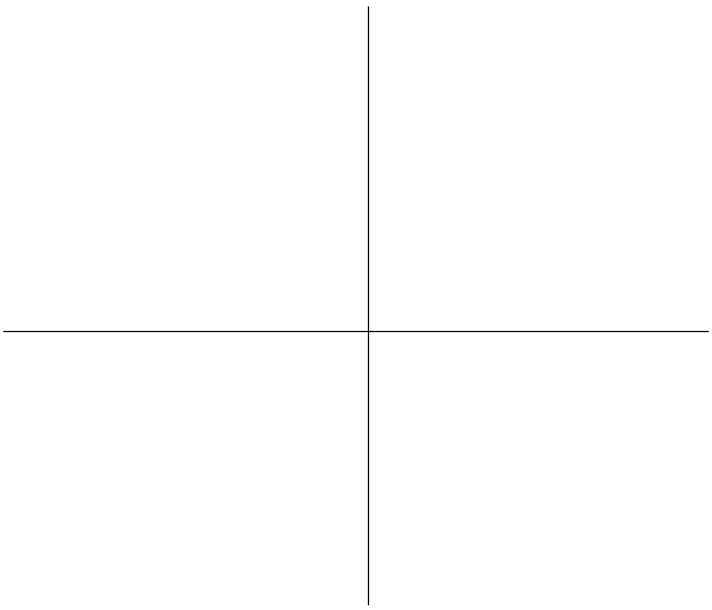
8 minutes

1. Express $x^2 - 11x + 37$ in the form $(x + a)^2 + b$ **(2)**

2. Express $4x^2 + 6x - 7$ in the form $a(x + b)^2 + c$ **(3)**

3. Solve the equation $2x^2 - 4x + 1 = 0$ by completing the square. **(3)**

4. Sketch the curve $y = 30 + 8x + x^2$ showing the exact coordinates of its turning point and the point where it crosses the y-axis. **(6)**



Solutions

1.

$x^2 - 11x + 37 = (x - \frac{11}{2})^2 - \frac{121}{4} + 37$	**M1**
$= (x - \frac{11}{2})^2 - \frac{27}{4}$	**M1**

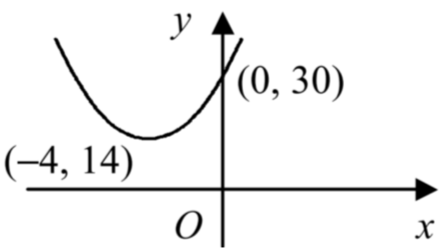
2.

$4x^2 + 6x - 7 = 4[x^2 + \frac{3}{2}x] - 7$	**M1**
$= 4[(x + \frac{3}{4})^2 - \frac{9}{16}] - 7$	**M1**
$= 4(x + \frac{3}{4})^2 - \frac{37}{4}$	**M1**

3.

$2x^2 + 8x + 5 = 0$ $y = 2[x^2 + 4x] + 5$	**M1**
$y = 2[(x + 2)^2 - 4] + 5$ $y = 2(x + 2)^2 - 3$	**M1**
Therefore, turning point is (-2, -3) and is a minimum (as coefficient of x^2 is positive)	**M1**

4.

$y = 30 + 8x + x^2 = x^2 + 8x + 30$	**M1**
$y = (x + 4)^2 - 16 + 30$	**M1**
$y = (x + 4)^2 + 14$	**M1**
Minimum at (-4, 14) When $x = 0$, $y = 30$	**M1**
	**M1** **(shape)** **M1** **(y-intercept)**

Topic: Functions Chapter Reference: Pure 1, Chapter 2	5 minutes

1. The functions f and g are defined as,

$$f(x) = x^2, \text{ for all real values of } x$$
$$g(x) = \frac{1}{x+2}, \text{ for all real values of } x,\ x \neq 2$$

a. Find $fg(x)$ **(1)**
b. Solve the equation $fg(x) = 4$ **(3)**

2. The functions f and g are defined with their respective domains by,

$$f(x) = \sqrt{x-2} \text{ for } \geq 2$$
$$g(x) = \frac{1}{x} \text{ for real values of } x,\ x \neq 0$$

a. Find $fg(x)$ **(1)**
b. Solve the equation $fg(x) = 1$ **(3)**

Solutions

1a.

$fg(x) = \frac{1}{(x+2)^2}$	**M1**

1b.

$\frac{1}{(x+2)^2} = 4$ $(x+2)^2 = \frac{1}{4}$	**M1**
$x + 2 = \pm\frac{1}{2}$	**M1**
$x = \frac{1}{2} - 2, x = -\frac{3}{2}$ $x = -\frac{1}{2} - 2, x = -\frac{5}{2}$	**M1**

2a.

$fg(x) = \sqrt{\frac{1}{x} - 2}$	**M1**

2b.

$\frac{1}{x} - 2 = 1$	**M1**
$\frac{1}{x} = 3$	**M1**
$x = \frac{1}{3}$	**M1**

A-Level Starter Activity

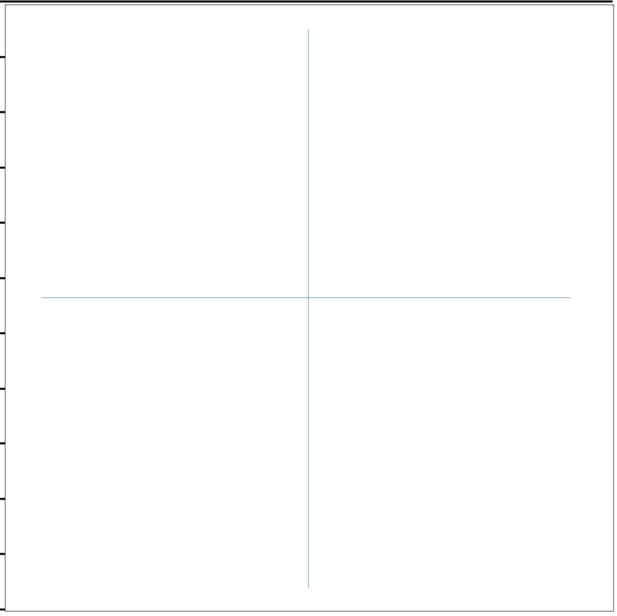
Jethwa Maths

Topic: Quadratic Graphs Chapter Reference: Pure 1, Chapter 2	8 minutes

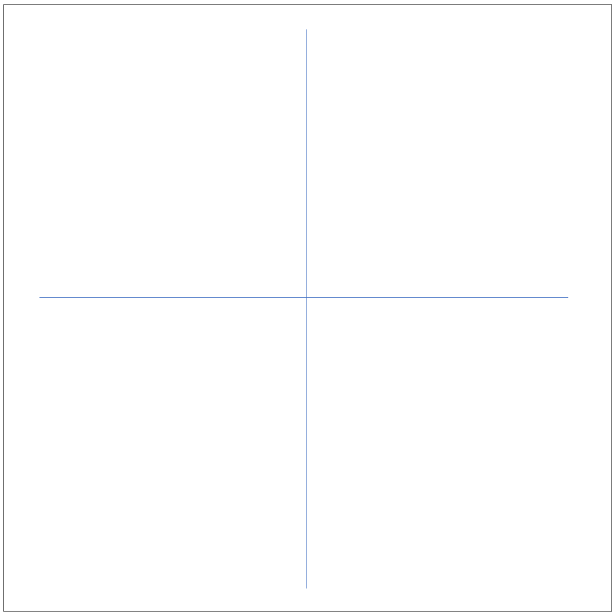
1a. Show that $x^2 + 6x + 11$ can be written as, $(x + p)^2 + q$, where p and q are integers to be found. **(1)**

b. In the space, sketch the curve with equation $y = x^2 + 6x + 11$, showing clearly any intersections with the coordinate axes. **(2)**

c. Find the value of the discriminant of $x^2 + 6x + 11$ **(2)**



2. On the aces below, sketch the graph of, $y = x(4 - x)$ **(2)**

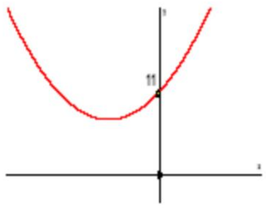


Solutions

1a.

$(x + 3)^2 + 2$	**M1**
$p = 3$, $q = 2$	

1b.



U shape with min in 2nd quad
(Must be above x-axis and not on y = axis)

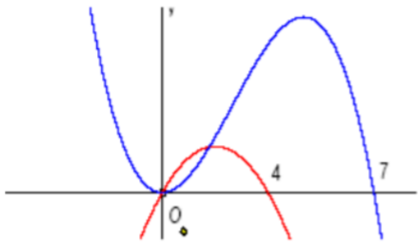
U shape crossing y-axis at (0, 11) only
(Condone (11,0) marked on y-axis)

1c.

$b^2 - 4ax = 6^2 - 4 \times 11$	**M1**
= -8	**M1**

2.

RED LINE ONLY



(i)	∩ shape (anywhere on diagram)	B1
	Passing through or stopping at (0, 0) and (4, 0) only(Needn't be ∩ shape)	B1

A-Level Starter Activity

Jethwa Maths

Topic: Solving Quadratic Equations Chapter Reference: Pure 1, Chapter 2	**8 minutes**

1. Solve the equation $x - 6x^{\frac{1}{2}} + 2 = 0$, giving your answers in the form $p \pm q\sqrt{r}$, where p, q and r are integers. **(2)**

2. Solve the equation $x – 4x^3$ **(3)**

3. Solve the equation $\frac{5}{2-x} + \frac{x-5}{x+2} + \frac{3x+8}{x^2-4} = 0$ **(4)**

<u>**Solutions**</u>

1.

$x - 6x^{\frac{1}{2}} + 2 = 0$ Square all terms, $x^2 - 36x + 4 = 0$	**M1**
$x = 16 + 6\sqrt{7}$ $x = 16 - 6\sqrt{7}$	**M1**

2.

$x - 4x^3 = x(1 - 4x^2) = 0$	**M1**
$x = 0$	**M1**
$1 - 4x^2 = 0$ $4x^2 = 1$ $x^2 = \frac{1}{4}$ $x = \pm\sqrt{\frac{1}{2}}$	**M1**

3.

$\frac{5}{2-x} + \frac{x-5}{x+2} + \frac{3x+8}{x^2-4} = 0$ $\frac{5(x+2)}{(2-x)(x+2)} + \frac{(x-5)(2-x)}{(x+2)(2-x)} + \frac{3x+8}{x^2-4} = 0$	**M1**
$\frac{3x+8}{x^2-4} = -\frac{3x+8}{4-x^2}$ $\frac{5(x+2)}{(2-x)(x+2)} + \frac{(x-5)(2-x)}{(x+2)(2-x)} - \frac{3x+8}{4-x^2} = 0$	**M1**
$5x + 10 + 2x - 10 - x^2 + 5x - 3x + 8 = 0$ $5x + 10 + 7x - 10 - x^2 - 3x - 8 = 0$ $x^2 - 9x + 8 = 0$	**M1**
$(x - 1)(x + 8) = 0$ $x = 1$ $x = -8$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: The Discriminant Chapter Reference: Pure 1, Chapter 2	8 minutes

1. The equation $(k + 3)x^2 + 6x + k = 5$, where k is a constant

Has two distinct real solutions for x.

a. Show that k satisfies, $k^2 - 2k - 24 < 0$ **(3)**

b. Hence find the set of possible values of k. **(3)**

2. The equation $x^2 + (k - 3)x + (3 - 2k) = 0$, where k is a constant, has two distinct real roots.

a. Show that k satisfies $k^2 + 2k - 3 > 0$ **(2)**

b. Find the set of possible values of k. **(3)**

Solutions

1a.

$(k + 3)x^2 + 6x + k = 5$ $(k + 3)x^2 + 6x + (k - 5) = 0$	**M1**
Two distinct solutions, $6^2 - 4(k + 3)(k - 5) > 0$ $36 - 4(k^2 - 2k - 16) > 0$ $36 - 4k^2 - 8k + 60 > 0$	**M1**
$k^2 - 2k - 24 < 0$	**M1**

1b.

$(k - 6)(k + 4) < 0$	**M1**
$k - 6 = 0$ $k = 6$ $k + 4 = 0$ $k = -4$	**M1**
As $y < 0$, $-6 < y < 4$	**M1**

2a.

$b^2 - 4ac > 0$ $(k - 3)^2 - (4)(1)(3 - 2k) > 0$	**M1**
$k^2 - 6k + 9 - 12 + 8k > 0$ $k^2 + 2k - 3 > 0$	**M1**

2b.

$k^2 + 2k - 3 > 0$ $(k + 3)(k - 1) > 0$	**M1**
$k = -3$ or $k = 1$	**M1**
As $y > 0$, $k < -3$ or $k > 1$	**M1**