Further Maths A-Level Starter Activity

Topic: The Method of Differences (1)

Chapter Reference: Core Pure 2, Chapter 2

10 minutes

1. Given that $\frac{r+1}{r+2} - \frac{r}{r+1} = \frac{1}{(r+1)(r+2)}$, find an expression, in terms of n, for **(4)**

$$\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \cdots + \frac{1}{(n+1)(n+2)}.$$

2. Given that $\frac{1}{r} - \frac{1}{r+2} = \frac{2}{r(r+2)}$, find an expression, in terms of n, for **(5)**

$$\frac{2}{1\times3} + \frac{2}{2\times4} + \cdots + \frac{2}{n(n+2)}.$$

Solutions

1. Either

$\frac{2}{3}-\frac{1}{2}+\frac{3}{4}-\frac{2}{3}\ \ldots\ \frac{n+1}{n+2}-\frac{n}{n+1}$ Express terms as differences	**M1**
At least first two and last term correct	**A1**
Show or imply that pairs of terms cancel	**M1**
$\frac{n+1}{n+2}-\frac{1}{2}$ Obtain correct answer in any form	**A1**

Or

State that $\sum_{r=1}^{n} u_r = f(n+1) - f(1)$	**M2**
Each term correct	**A1A1**

2.

Express terms as differences	**M1**
Express 1st 3 (or last 3) terms so that cancelling occurs	**M1**
Obtain $1+\frac{1}{2}$	**A1**
Obtain $-\frac{1}{n+2}$, $-\frac{1}{n+1}$	**A1**
$\frac{3}{2}-\frac{1}{n+1}-\frac{1}{n+2}$ Obtain correct answer in any form	**A1**

Further Maths A-Level Starter Activity

Topic: Higher Derivatives (2)

Chapter Reference: Core Pure 2, Chapter 2

10 minutes

1. $y = x^3 e^{2x}$. Find:

   a. $\frac{dy}{dx}$ **(2)**

   b. $\frac{d^2y}{dx^2}$ **(2)**

   c. $\frac{d^3y}{dx^3}$ **(2)**

   d. $\frac{d^4y}{dx^4}$ **(2)**

   e. $\frac{d^5y}{dx^5}$ **(2)**

Solutions

1.

a	$= (2x + 1)e^{2x}$	**A2**
b	$= (4x + 4)e^{2x}$	**A2**
c	$= (8x + 12)e^{2x}$	**A2**
d	$= (16x + 32)e^{2x}$	**A2**
e	$= (32x + 80)e^{2x}$	**A2**

Further Maths A-Level Starter Activity

Topic: Maclaurin Series (3)

Chapter Reference: Core Pure 2, Chapter 2

10 minutes

1. It is given that $f'(x) = \tanh^{-1}(\frac{1-x}{2+x})$, for $x > -\frac{1}{2}$.

   a. Show that $f'(x) = -\frac{1}{1+2x}$, and find $f''(x)$. **(6)**

   b. Show that the first three terms of the Maclaurin series for $f(x)$ can be written as $\ln a + bx + cx^2$, for constants a, b and c to be found. **(4)**

Solutions

1a.

Attempt quotient/product on bracket	**M1**
Get $-3/(2+x)^2$ May be implied	**A1**
Use Formulae Booklet or derive from tanh $y = (1\text{-}x)/(2+x)$ Attempt $\tanh^{-1}$ part in terms of x	**M1**
Get $\dfrac{-3}{(2+x)^2}\cdot\dfrac{1}{1-((1-x)/(2+x))^2}$	**A1**
Clearly tidy to A.G.	**A1**
Get $f''(x) = 2/(1+2x)^2$	**B1**

OR

Use reasonable ln definition	**M1**
Get $y=\frac{1}{2}\ln((1\text{-}k)/(1+k))$ for $k=(1\text{-}x)/(1+2x)$.	**A1**
Tidy to $y=\frac{1}{2}\ln(3/(1+2x)$	**A1**
Attempt chain rule	**M1**
Clearly tidy to A.G.	**A1**
Get $f''(x)$	**B1**

1b.

Attempt f(0), f ′(0) and f ″(0)	**M1**
Get $\tanh^{-1}\frac{1}{2}$, -1 and 2	**A1**
Replace $\tanh^{-1}\frac{1}{2} = \frac{1}{2}\ln 3$ $(=\ln\sqrt{3})$	**B1**
Get $\ln\sqrt{3} - x + x^2$	**A1**

Further Maths A-Level Starter Activity

Topic: Series Expansions of Compound Functions (4)

Chapter Reference: Core Pure 2, Chapter 2

10 minutes

1. Use the Maclaurin series, together with a suitable substitution, to show that:
   a. $(1-3x)\ln(1+2x) = 2x - 8x^2 + \frac{26}{3}x^3 - 12x^4 + \cdots$ **(3)**
   b. $e^{2x}\sin x = x + 2x^2 + \frac{11}{6x^3} + x^4 + \cdots$ **(3)**
   c. $\sqrt{1+x^2}e^{-x} = 1 - x + x^2 - \frac{2}{3}x^3 + \frac{1}{6}x^4 + \cdots$ **(3)**

Solutions

1a.

Substitution of $2x$	**A1**
Use of standard Maclaurin expansion formula	**M1**
$2x - 8x^2 + \frac{26}{3}x^3 - 12x^4 + \cdots$	**A1**

1b.

Substitution of $2x$	**A1**
Use of standard Maclaurin expansion formula	**M1**
$x + 2x^2 + \frac{11}{6}x^3 + x^4 + \cdots$	**A1**

1c.

Substitution of x^2	**A1**
Use of standard binomial expansion formula	**M1**
$1 - x + x^2 - \frac{2}{3}x^3 + \frac{1}{6}x^4 + \cdots$	**A1**

Topic: Series Expansions of Compound Functions (5)

Chapter Reference: Core Pure 2, Chapter 2

10 minutes

1. Given that $f(x) = e^{sinx}$,

   a. Find $f'(0)$ and $f''(0)$. **(4)**

   b. Hence find the first three terms of the Maclaurin series for $f(x)$. **(2)**

Solutions

1a.

Reasonable attempt at chain rule	**M1**
Reasonable attempt at product/quotient rule	**M1**
Correctly get f ′(0) =1	**A1**
Correctly get f″(0) = 1	**A1**

1b.

Reasonable attempt at Maclaurin with their values	**M1**
Get $1 + x + \frac{1}{2}x^2$	**A1**