Further Maths A-Level Starter Activity

Topic: Poisson Distribution (1)

Chapter Reference: Further Statistics 1, Chapter 2

9 minutes

1. The number of telephone calls per day, X, received by Candice may be modelled by a Poisson distribution with mean 3.5.

   The number of emails per day, Y, received by Candice may be modelled by a Poisson distribution with mean 6.0.

   For any particular day, find:

a. $P(X = 3)$ **(2)**

b. $P(Y \geq 5)$. **(2)**

2. In a village, power cuts occur randomly at a rate of 3 per year.

   Find the probability that in any given year there will be

a. Exactly 7 power cuts,

b. At least 4 power cuts. **(5)**

Solutions

1a.

$P(X = 3) = \frac{e^{-3.5} \times (3.5)^3}{3!}$	**M1**
= 0.216	**A1**

1b.

$P(Y \geq 5) = 1 - P(Y \leq 4)$ = 1 – 0.2851	**M1**
= 0.715	**A1**

2a.

Let X be the random variable the number power cuts. $X \sim Po(3)$	**B1**
$P(X = 7) = P(X \leq 7) - P(X \leq 6)$ or $\frac{e^{-3} \times 3^7}{7!}$ = 0.9881 – 0.9665	**M1**
= 0.0216 awrt 0.0216	**A1**

2b.

b. $P(X \geq 4) = 1 - P(X \leq 3)$ = 1 – 0.6472	**M1**
= 0.3528 awrt 0.353	**A1**

Further Maths A-Level Starter Activity

Topic: Poisson Distribution (2)

Chapter Reference: Further Statistics 1, Chapter 2

7 minutes

1. The number of A-grades, X, achieved in total by students at Lowkey School in their Mathematics examinations each year can be modelled by a Poisson distribution with a mean of 3.

a. Determine the probability that, during a 5-year period, students at Lowkey School achieve a total of more than 18 A-grades in their Mathematics examinations. **(3)**

b. The number of A-grades, Y, achieved in total by students at Lowkey School in their English examinations each year can be modelled by a Poisson distribution with a mean of 7.

i. Determine the probability that, during a year, students at Lowkey School achieve a total of fewer than 15 A-grades in their Mathematics and English examinations. **(3)**

ii. What assumptions did you make in answering part b.i.? **(1)**

Solutions

1a.

For a 1-year period The number of A grades ~ Po(3)	
For a 5-year period The number of A grades ~ Po(15)	**B1**
P(Total A-grades > 18) = 1 – (Total ≤ 18) = 1 – 0.8195	**M1**
= 0.181 WFW 0.180 to 0.181	**A1**

1bi.

$X + Y \sim \text{Po}(10)$	**B1**
$P(X + Y \leq 14) = 0.917$ AWFW 0.916 to 0.917	**M1 A1**

1bii.

X and Y are independent variables.	**E1**

Topic: Poisson Distribution (3)

Chapter Reference: Further Statistics 1, Chapter 2

9 minutes

1. A botanist is studying the distribution of daisies in a field. The field is divided into a number of equal sized squares. The mean number of daisies per square is assumed to be 3. The daisies are distributed randomly throughout the field.

   Find the probability that, in a randomly chosen square there will be

a. More than 2 daisies, **(3)**

b. Either 5 or 6 daisies. **(2)**

2. The botanist decides to count the number of daisies, *x*, in each of 80 randomly selected squares within the field. The results are summarised below

$$\sum x = 295 \qquad \sum x^2 = 1386$$

a. Calculate the mean and the variance of the number of daisies per square for the 80 squares. Give your answers to 2 decimal places. **(3)**

b. Explain how the answers from part c. support the choice of a Poisson distribution as a model. **(1)**

Solutions

1a.

The random variable X is the number of daisies in a square. Poisson(3)	**B1**
$1 - P(X \leq 2) = 1 - 0.4232$ or $1 - e^{-3}(1 + 3 + \frac{3^2}{2!})$	**M1**
$= 0.5768$	**A1**

1b.

$P(X \leq 6) - P(X \leq 4) = 0.9665 - 0.8153$ or $e^{-3}(\frac{3^5}{5!} + \frac{3^6}{6!})$	**M1**
$= 0.1512$	**A1**

2a.

$\mu = 295 \div 80$ $= 3.69$	**B1**
$\text{Var}(X) = \frac{1386}{80} - (\frac{295}{80})^2$	**M1**
$= 3.73/3.72/3.71$ accept $s^2 = 3.77$	**A1**

2b.

For a Poisson model, Mean = Variance; For these data $3.69 \approx 3.73$ $\Rightarrow$Poisson model	**B1**

Topic: Poisson Distribution (4)

Chapter Reference: Further Statistics 1, Chapter 2

9 minutes

1. Connie and Angela play each other at badminton and for each game, independently of all others, the probability that Connie loses is 0.2.

   Find the probability that, in 9 games, Connie loses

a. Exactly 3 of the games, **(3)**

b. Fewer than half of the games. **(2)**

Connie attends coaching sessions for 2 months. After completing the coaching, the probability that he loses each game, independently of all others, is 0.05.

Connie and Angela agree to play a further 60 games.

c. Calculate the mean and variance for the number of these 60 games that Connie loses. **(2)**

d. Using a suitable approximation calculate the probability that Connie loses more than 4 games. **(3)**

Solutions

1a.

Let X be the random variable the number of games Connie loses. $X \sim B(9, 0.2)$	**B1**
$P(X \le 3) - P(X \le 2) = 0.9144 - 0.7382$ or $(0.2)^3(0.8)^6\frac{9!}{3!6!}$	**M1**
$= 0.1762$ awrt 0.176	**A1**

1b.

$P(X \le 4) = 0.9804$ awrt 0.98	**M1 A1**

1c.

Mean $= 60 \times 0.05$ $= 3$	**M1**
Variance $= 60 \times 0.05 \times (1 - 0.05)$ $= 2.85, \frac{57}{20}$ oe	**M1**

1d.

Po(3) (Poisson)	**M1**
$P(X > 4) = 1 - P(X \le 4)$ $= 1 - 0.8153$	**M1**
$= 0.1847$	**A1**

Further Maths A-Level Starter Activity

Topic: Poisson Distribution (5)

Chapter Reference: Further Statistics 1, Chapter 2

10 minutes

1. The number of defects per metre in a roll of cloth has a Poisson distribution with mean 0.25.

Find the probability that

a. A randomly chosen metre of cloth has 1 defect, **(2)**

b. The total number of defects in a randomly chosen 6 metre length of cloth is more than 2. **(3)**

A tailor buys 300 metres of cloth.

c. Using a suitable approximation find the probability that the tailor's cloth will contain less than 90 defects. **(5)**

<u>**Solutions**</u>

1a.

P(X = 1) = $0.25e^{-0.25}$	**M1**
= 0.1947 awrt 0.195	**A1**

1b.

X ~ Po(1.5)	**B1**
P(X > 2) = 1 – P(X ≤ 2) = 1 – 0.8088	**M1**
= 0.1912 awrt 0.191	**A1**

1c.

[λ = 300 × 0.25 = 75] X ~ N(75,75) For normal approximation and correct mean Var (X) = 75or sd = √75 or awrt 8.66 (may be given if correct in standardisation formula)	**B1** **B1**
P(X < 90) = P($X \leq \frac{89.5-75}{\sqrt{75}}$ = P(Z ≤ 1.6743...) Using either 89.5 or 88.5 Standardising using their mean and their sd, using [89.5, 88.5 or 89] and for finding correct area	**M1** **M1**
= awrt 0.953 or 0.952	**A1**
NB use of Poisson gives an answer of 0.9498 and gains no marks	