

**Further Maths
A-Level Starter
Activity**



Topic: Argand Diagrams (1)

Chapter Reference: Core Pure 1, Chapter 2

**10
minutes**

1. The complex numbers z_1 and z_2 are given by $z_1 = 2 - i$ and $z_2 = -8 + 9i$.

Show z_1 and z_2 on a single Argand diagram.

(1)

2. Find the roots of the equation $z^2 + 2z + 17 = 0$, giving your answers in the form $a + ib$, where a and b are integers and show these roots on a single Argand diagram.

(4)

3. Given that $z_1 = 1 + i$ and $z_2 = \frac{6+2i}{z_1}$,

a) Find z_2 in the form $a + ib$, where a and b are real.

(2)

b) Show on an Argand diagram the point P representing z_1 and the point Q representing z_2 .

(2)

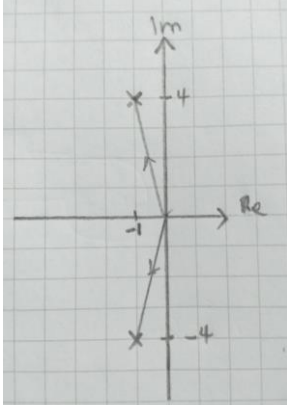


Solutions

1.

	M1
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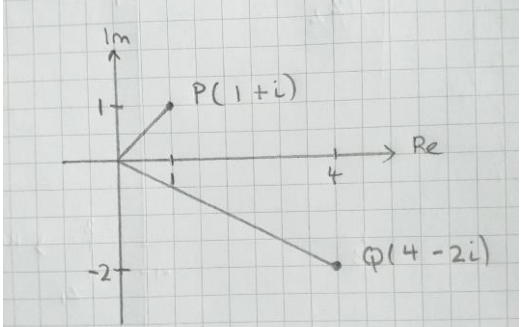
2.

$(z + 1)^2 - 1 + 17 = 0$ $(z + 1)^2 = -16$ $z + 1 = \pm\sqrt{16}$ $z = -1 \pm\sqrt{16}$	M1
$z = -1 + 4i$	M1
$z = -1 - 4i$	M1
	M1 must be conjugate pair

3a.

$\frac{6+2i}{1+i} \times \frac{1-i}{1-i} = \frac{6-6i+2i+2}{1+1} = \frac{8-4i}{2}$	M1
$= 4 - 2i$	M1

3b

	M1 correct position of P M1 correct position of Q
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**Further Maths
A-Level Starter
Activity**



Topic: Modulus and Argument (2)

Chapter Reference: Core Pure 1, Chapter 3

**10
minutes**

1. Given that $z = -3 + 4i$,

- a) Find the modulus of z , (2)
- b) The argument of z in radians to 2 decimal places. (2)

2.

$$z = -5 - 12i$$

Find, showing your working,

- a) The value of $|z|$, (2)
- b) The value of $\arg(z)$, giving your answer in radians to 2 decimal places. (2)
- c) Show z and $y = 2 - 3i$ on a single Argand diagram. (1)



Solutions

1a.

$ z = \sqrt{3^2 + 4^2}$	M1
$ z = 5$	M1

1b.

$\arg z = \pi - \arctan \frac{4}{3}$ (oe)	M1
$\arg z = 2.21$	M1

2a.

a) $ z = \sqrt{(-5)^2 + (-12)^2}$	M1
$= 13$	M1

2b.

$\arg z = -(\pi - \arctan \frac{12}{5})$ (oe)	M1
$= -1.97$ (or 4.32) allow awrt	M1

2c.

	M1
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1. Express $z = \sqrt{3} + i$ in modulus and argument form. (2)

2.

$$z = 4\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right), \text{ and } w = 3\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right).$$

- Express zw in the form $r(\cos \theta + i \sin \theta)$, $r > 0$, $-\pi < \theta < \pi$. (3)

3. Given that $z = 4\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)$ and $w = 1 - i\sqrt{3}$,

- Find $\arg\left(\frac{z}{w}\right)$. (3)



Solutions

1.

$\begin{aligned} z &= \sqrt{3 + 1} \\ &= 2 \\ \arg(z) &= \arctan \frac{1}{\sqrt{3}} \\ &= \frac{\pi}{6} \end{aligned}$	M1
$z = 2\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)$	M1

2.

$\begin{aligned} zw &= 4 \times 3 \\ &= 12 \end{aligned}$	M1
$\begin{aligned} \arg(zw) &= \frac{\pi}{4} + \frac{2\pi}{3} \\ &= \frac{11\pi}{12} \end{aligned}$	M1
$zw = 12\left(\cos \frac{11\pi}{12} + i \sin \frac{11\pi}{12}\right)$	M1

3.

$\begin{aligned} \arg(w) &= -\left(\arctan \frac{\sqrt{3}}{1}\right) \\ &= -\frac{\pi}{3} \end{aligned}$	M1
$\arg(z) - \arg(w) = \frac{3\pi}{4} - \left(-\frac{\pi}{3}\right) = \frac{13\pi}{12}$	M1
$\arg\left(\frac{z}{w}\right) = -\frac{11\pi}{12}$	M1





1. Sketch, on an Argand diagram, the locus given by $|z - 1 + i| = \sqrt{2}$. (3)

2. In an Argand diagram the loci C_1 and C_2 are given by

$$|z| = 2 \text{ and } \arg(z) = \frac{1}{3}\pi$$

respectively.

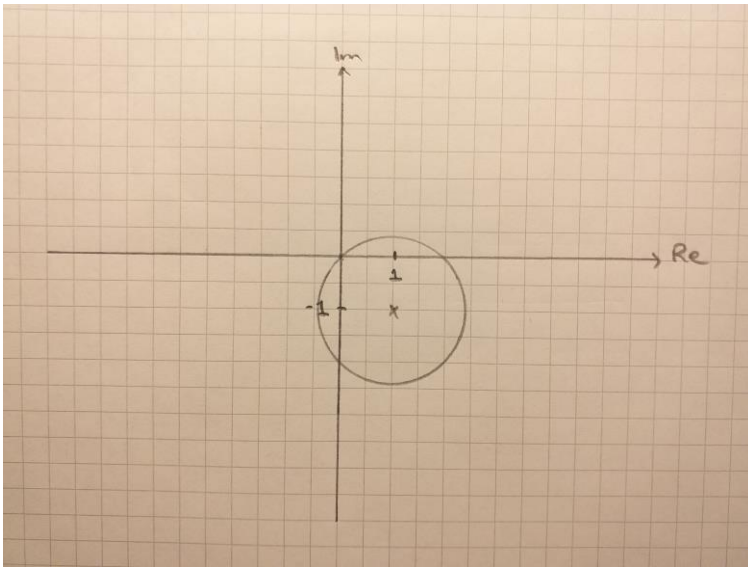
a) Sketch, on a single Argand diagram, the loci C_1 and C_2 . (5)

b) Hence find, in the form $x + iy$, the complex number representing the point of intersection of C_1 and C_2 . (2)



Solutions

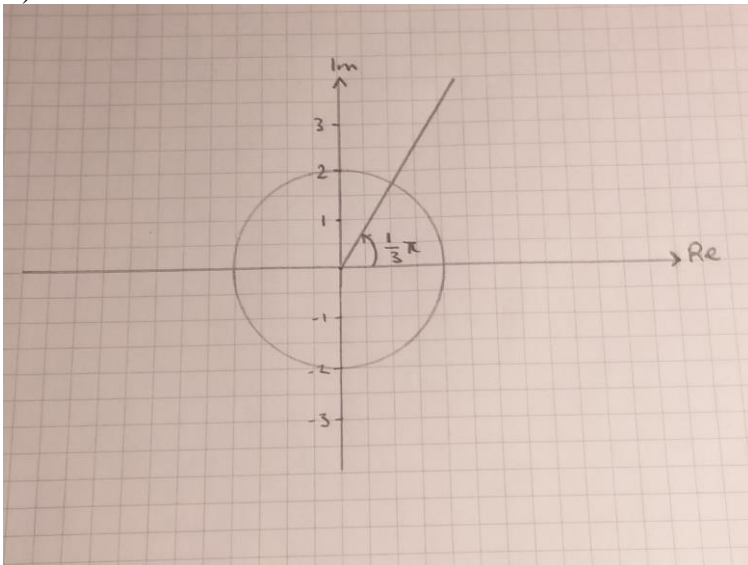
1.



M1 Circle
M1 Centre (1, -1)
M1 Passing through (0, 0)

2.

a)



M1 Circle
M1 Centre O radius 2
M1 One straight line
M1 Through O with +ve slope
M1 In 1st quadrant only

b) Real number = $2\cos\left(\frac{\pi}{3}\right) = 1$
 Imaginary number = $2\sin\left(\frac{\pi}{3}\right) = \sqrt{3}$
 $= 1 + i\sqrt{3}$

M1

M1





1. Shade in, on an Argand diagram, the region represented by $0 \leq \arg(z - 2 - 2i) \leq \frac{\pi}{4}$. (3)

2.

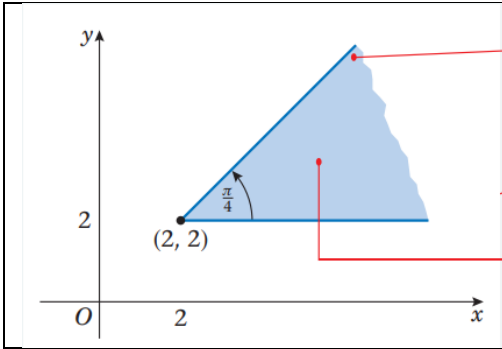
a) Sketch, on an Argand diagram, the locus given by $|z - 3 + 3i| = 3\sqrt{2}$. (3)

b) Shade on your diagram the region given by $2 \leq |z - 3 + 3i| \leq 3\sqrt{2}$. (3)



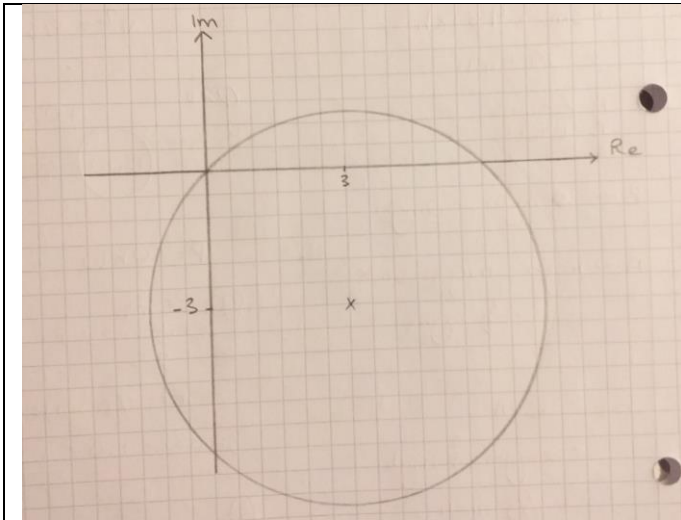
Solutions

1.



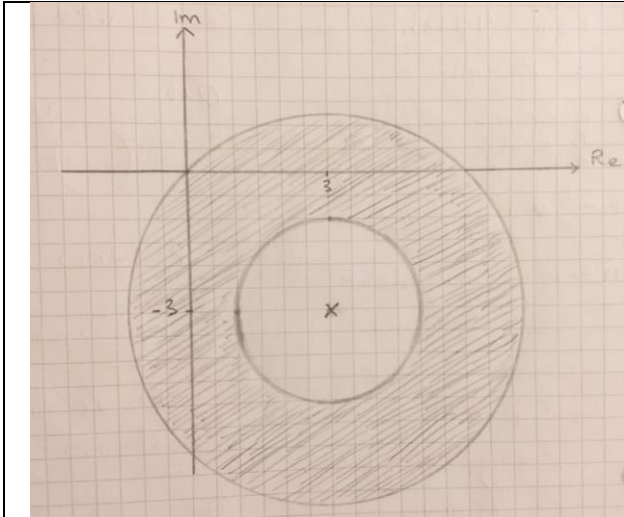
M1 half lines coming from point (2,2)
M1 two half lines drawn correctly as shown
M1 shaded correctly as shown

2a.



M1 Circle
M1 Centre (3, -3)
M1 Passing through (0, 0)

2b.



M1 Sketch a concentric circle
M1 Inside (a) and not touching axes
M1 Shade between the circles