Further Maths A-Level Starter Activity

Topic: Argand Diagrams (1) Chapter Reference: Core Pure 1, Chapter 2	10 minutes

1. The complex numbers z_1 and z_2 are given by $z_1 = 2 - i$ and $z_2 = -8 + 9i$.
   Show z_1 and z_2 on a single Argand diagram. **(1)**

2. Find the roots of the equation $z^2 + 2z + 17 = 0$, giving your answers in the form $a + ib$, where a and b are integers and show these roots on a single Argand diagram. **(4)**

3. Given that $z_1 = 1 + i$ and $z_2 = \frac{6+2i}{z_1}$,

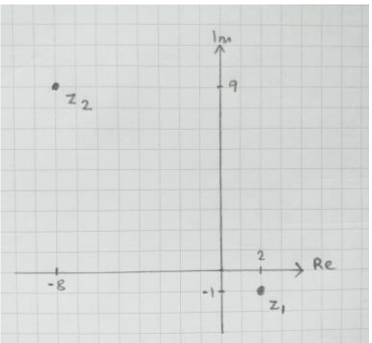
a) Find z_2 in the form $a + ib$, where a and b are real. **(2)**

b) Show on an Argand diagram the point P representing z_1 and the point Q representing z_2. **(2)**

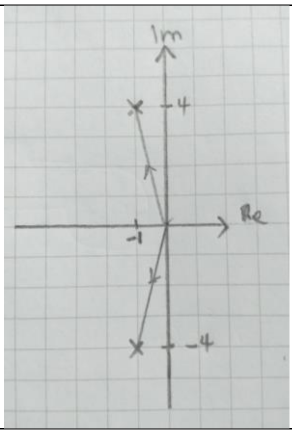
Content created by Uwaila R. Ehioghae for JethwaMaths

Solutions

1.

	**M1**

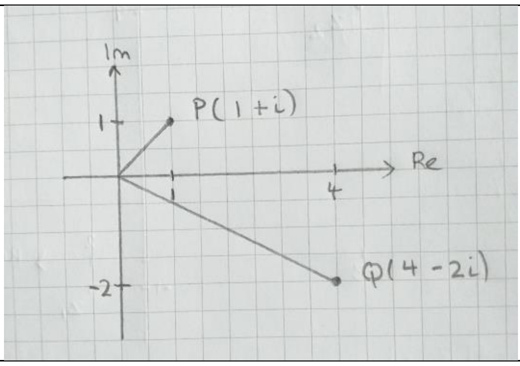
2.

$(z + 1)^2 - 1 + 17 = 0$ $(z + 1)^2 = -16$ $z + 1 = \pm\sqrt{16}$ $z = -1 \pm\sqrt{16}$	**M1**
$z = -1 + 4i$	**M1**
$z = -1 - 4i$	**M1**
	**M1** must be conjugate pair

3a.

$\frac{6+2i}{1+i}$ $\frac{6+2i}{1+i} \times \frac{1-i}{1-i} = \frac{6-6i+2i+2}{1+1} = \frac{8-4i}{2}$	**M1**
$= 4 - 2i$	**M1**

3b

	**M1** correct position of P **M1** correct position of Q

Further Maths A-Level Starter Activity

Topic: Modulus and Argument (2)

Chapter Reference: Core Pure 1, Chapter 3

10 minutes

1. Given that $z = -3 + 4i$,

a) Find the modulus of z, **(2)**

b) The argument of z in radians to 2 decimal places. **(2)**

2.

$$z = -5 - 12i$$

Find, showing your working,

a) The value of $|z|$, **(2)**

b) The value of arg (z), giving your answer in radians to 2 decimal places. **(2)**

c) Show z and $y = 2 - 3i$ on a single Argand diagram. **(1)**

Solutions

1a.

$\|z\| = \sqrt{3^2 + 4^2)}$	**M1**
$\|z\| = 5$	**M1**

1b.

arg $z = \pi - \arctan\frac{4}{3}$ (oe)	**M1**
arg z = 2.21	**M1**

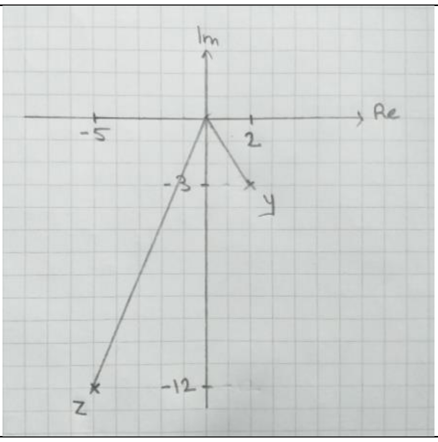
2a.

a) $\|z\| = \sqrt{(-5)^2 + (-12)^2)}$	**M1**
$= 13$	**M1**

2b.

arg $z = -(\pi - \arctan\frac{12}{5})$ (oe)	**M1**
$= -1.97$ (or 4.32) allow awrt	**M1**

2c.

	**M1**

Further Maths A-Level Starter Activity

Topic: Modulus-Argument Form of Complex Numbers (3)

Chapter Reference: Core Pure 1, Chapter 2

8 minutes

1. Express $z = \sqrt{3} + i$ in modulus and argument form. **(2)**

2.

$$z = 4\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right), \text{ and } w = 3\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right).$$

Express zw in the form $r(\cos\theta + i\sin\theta)$, $r > 0$, $-\pi < \theta < \pi$. **(3)**

3. Given that $z = 4\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$ and $w = 1 - i\sqrt{3}$,

Find $\arg\left(\frac{z}{w}\right)$. **(3)**

Solutions

1.

$\|z\| = \sqrt{3+1}$ $= 2$ arg (z) = arctan $\frac{1}{\sqrt{3}}$ $= \frac{\pi}{6}$	**M1**
$z = 2(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$	**M1**

2.

$\|zw\| = 4 \times 3$ $= 12$	**M1**
arg $(zw) = \frac{\pi}{4} + \frac{2\pi}{3}$ $= \frac{11\pi}{12}$	**M1**
$zw = 12(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12})$	**M1**

3.

arg$(w) = -($arctan$\frac{\sqrt{3}}{1})$ $= -\frac{\pi}{3}$	**M1**
arg(z) – arg$(w) = \frac{3\pi}{4} - (-\frac{\pi}{3}) = \frac{13\pi}{12}$	**M1**
arg$(\frac{z}{w}) = -\frac{11\pi}{12}$	**M1**

Further Maths A-Level Starter Activity

Topic: Argand Diagram and Loci (4)

Chapter Reference: Core Pure 1, Chapter 2

10 minutes

1. Sketch, on an Argand diagram, the locus given by $|z - 1 + i| = \sqrt{2}$. **(3)**

2. In an Argand diagram the loci C_1 and C_2 are given by

$$|z| = 2 \text{ and } \arg(z) = \frac{1}{3}\pi$$

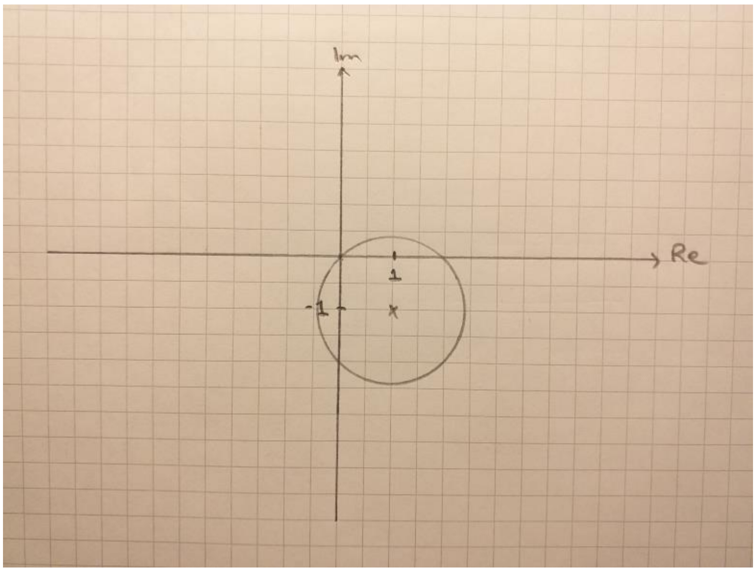
respectively.

a) Sketch, on a single Argand diagram, the loci C_1 and C_2. **(5)**

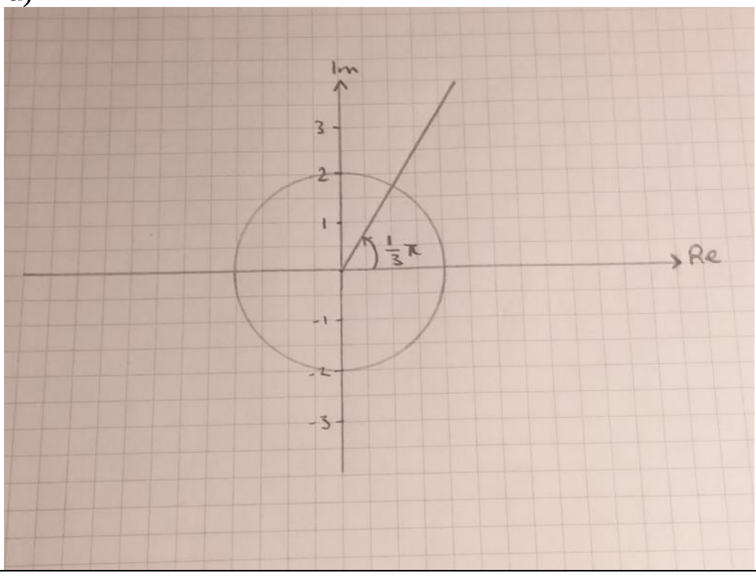
b) Hence find, in the form $x + iy$, the complex number representing the point of intersection of C_1 and C_2. **(2)**

Solutions

1.

	**M1** Circle **M1** Centre (1, -1) **M1** Passing through (0, 0)

2.

a) 	**M1** Circle **M1** Centre *O* radius 2 **M1** One straight line **M1** Through *O* with +ve slope **M1** In 1st quadrant only
b) Real number = $2\cos(\frac{\pi}{3}) = 1$ Imaginary number = $2\sin(\frac{\pi}{3}) = \sqrt{3}$	**M1**
$= 1 + i\sqrt{3}$	**M1**

Topic: Argand Diagrams and Regions (5)

Chapter Reference: Core Pure 1, Chapter 2

9 minutes

1. Shade in, on an Argand diagram, the region represented by $0 \leq \arg(z - 2 - 2i) \leq \frac{\pi}{4}$. **(3)**

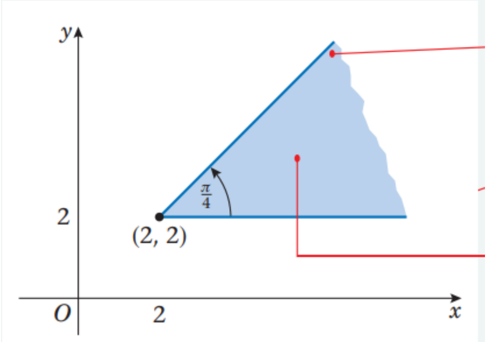
2.

a) Sketch, on an Argand diagram, the locus given by $|z - 3 + 3i| = 3\sqrt{2}$. **(3)**

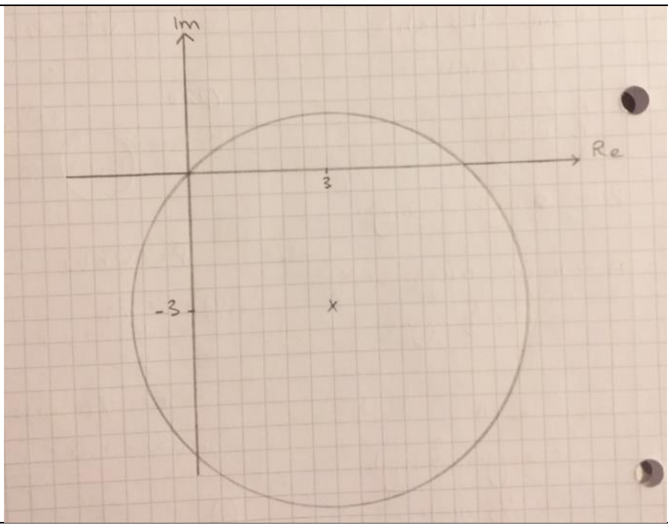
b) Shade on your diagram the region given by $2 \leq |z - 3 + 3i| \leq 3\sqrt{2}$. **(3)**

Solutions

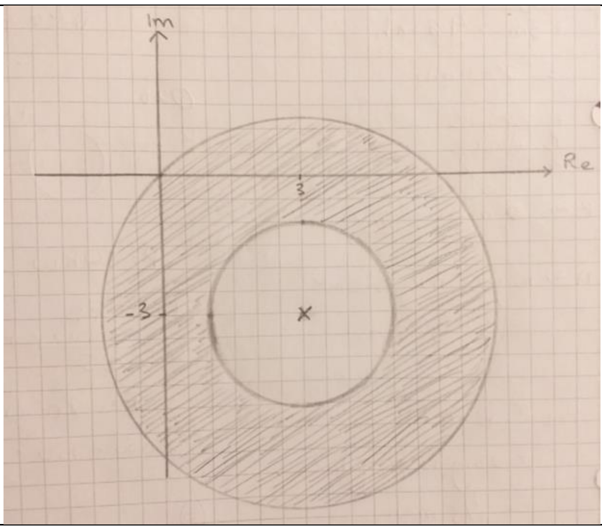
1.

	**M1** half lines coming from point (2,2) **M1** two half lines drawn correctly as shown **M1** shaded correctly as shown

2a.

	**M1** Circle **M1** Centre (3, -3) **M1** Passing through (0, 0)

2b.

	**M1**Sketch a concentric circle **M1** Inside (a) and not touching axes **M1** Shade between the circles