Further Maths A-Level Starter Activity

Topic: Expected Value of a Discrete Random Variable (1)

Chapter Reference: Further Statistics 1, Chapter 1

8 minutes

1. Part of the probability distribution of a variable, X, is given in the table.

x	0	1	2	3
$P(X = x)$		$\frac{3}{10}$	$\frac{1}{5}$	$\frac{2}{5}$

a. Find $P(X = 0)$. **(2)**

b. Find $E(X)$. **(2)**

2. The probability distribution of a random variable X is shown in the table.

x	1	2	3	4
$P(X = x)$	0.1	0.3	$2p$	p

a. Find p. **(2)**

b. Find $E(X)$. **(2)**

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Solutions

1a.

$1-(\frac{3}{10}+\frac{1}{5}+\frac{2}{5})$	**M1**
$\frac{1}{10}$	**M1**

1b.

$\frac{3}{10}+(2\times\frac{1}{5})+(3\times\frac{2}{5})$	**M1**
$\frac{19}{10}$ (oe)	**M1**

2a.

$0.1 + 0.3 + 2p + p = 1$	**M1**
$p = 0.2$ (oe)	**M1**

2b.

$(1 \times 0.1) + (2 \times 0.3) + (3 \times 0.8) + (4 \times 0.2)$	**M1**
$= 2.7$ (oe)	**M1**

Topic: Variance of a Discrete Random Variable (2)

Chapter Reference: Further Statistics 1, Chapter 1

8 minutes

1. The table shows the probability distribution for a random variable X.

x	0	1	2	3
$P(X = x)$	0.1	0.2	0.3	0.4

Calculate $E(X)$ and $\text{Var}(X)$. **(5)**

2. The probability distribution of X is given in the table.

x	0	1	2	3	4
$P(X = x)$	0.49	0.28	0.18	0.04	0.01

Calculate $E(X)$ and $\text{Var}(X)$. **(5)**

Solutions

1.

$E(X) = (0\times0.1) + (1\times0.2) + (2\times0.3) + (3\times0.4)$	**M1**
$E(X) = 2(.0)$	**A1**
$Var(X) = (02 \times0.1) +(1\times0.2) + (2^2 \times0.3) + (3^2 \times0.4)$ $(= 5)$	**M1**
$Var(X) = 5 – 2^2$	**M1**
$Var(X) = 1$	**A1**

2.

$E(X) = (1\times 0.28) + (2\times0.18) + (3\times0.04) + (4\times0.01)$	**M1**
$E(X) = 0.8$ (oe)	**A1**
$Var(X) = 0.28 + (2^2 \times0.18) + (3^2 \times0.04) + (4^2 \times0.01)$ $(= 1.52)$	**M1**
$Var(X) = 1.52 – 0.8^2$	**M1**
$Var(X) = 0.88$ (oe)	**A1**

Topic: Expected Value and Variance of a Function of X (3)

Chapter Reference: Further Statistics 1, Chapter 1

8 minutes

1. The table shows the probability distribution for a random variable X.

x	0	1	2	3
$P(X = x)$	0.3	0.2	0.1	0.4

Find

a. $E(X)$ **(2)**

b. Find $E(3X - 2)$ **(2)**

c. Show that the $Var(X) = 1.64$ **(3)**

d. Calculate $Var(3X - 2)$. **(2)**

Solutions

1a.

$E(X) = (1 \times 0.2) + (2 \times 0.1) + (3 \times 0.4)$	**M1**
$E(X) = 1.6$	**A1**

1b.

$E(3X - 2) = 3E(X) - 2$	**M0**
$E(3X - 2) = 3 \times 1.6 - 2$	**M1**
$E(3X - 2) = 2.8$	**A1**

1c.

$E(X^2) = 1 \times 0.2 + 4 \times 0.1 + 9 \times 0.4$ (= 4.2)	**M1**
$\text{Var}(X) = 4.2 - 1.6^2$	**M1**
$\text{Var}(X) = 1.64$	**A1**

1d.

$\text{Var}(aX + b) = a^2\text{Var}(X)$ $\text{Var}(3X - 2) = 3^2\ \text{Var}(X)$	**M1**
$\text{Var}(3X - 2) = 14.76$ (awrt 14.8)	**A1**

Topic: Problems Involving Random Variables (4)

Chapter Reference: Further Statistics 1, Chapter 1

9 minutes

1. The discrete random variable X has probability function

$$\mathrm{P}(X = x) = \begin{cases} k(2 - x), & x = 0,1,2, \\ k(x - 2), & x = 3 \\ 0, & \text{otherwise,} \end{cases}$$

where k is a positive constant.

a. Show that $k = 0.25$. **(2)**

b. Find $\mathrm{E}(X)$ and show that $\mathrm{E}(X^2) = 2.5$. **(4)**

c. Find $\mathrm{Var}(3X - 2)$. **(3)**

Solutions

1a.

$2k + k + 0 + k = 1$	**M1**
$4k = 1$ $k = 0.25$	**A1**

1b.

x	0	1	2	3	**M0**
$P(X = x)$	0.5	0.25	0	0.25	
$xP(X = x)$	0	0.25	0	0.75	
$x^2P(X = x)$	0	0.25	0	2.75	
$E(X) = \Sigma xP(X = x) = 0 + 0.25 + 0 + 0.75$					**M1**
$E(X) = 1$					**A1**
$E(X^2) = 0 + 0.25 + 0 + 2.25$					**M1**
$E(X^2) = 2.5$					**A1**

1c.

$Var(3X - 2) = 3^2 \; Var(X)$	**M1**
$Var(3X - 2) = 9(2.5 - 1^2)$	**M1**
$Var(3X - 2) = 13.5$	**A1**

Topic: Problems Involving Random Variables (5)

Chapter Reference: Further Statistics 1, Chapter 1

10 minutes

1. A fairground game involves trying to hit a moving target with a gunshot. A round consists of up to 3 shots. Ten points are scored if a player hits the target, but the round is over if the player misses. Linda has a constant probability of 0.6 of hitting the target and shots are independent of one another.
   The random variable X is the number of points Linda scores in a round.

a. Find the probability distribution of X. **(5)**

b. Find the mean and the standard deviation of X. **(5)**

Solutions

1a.

x	0	10	20	30	**M1 (*x* values)**
	0.4	0.6×0.4	$0.6^2 \times 0.4$	0.6^3	
$P(x = x)$	0.4	0.24	0.144	0.216	**M1** $P(X = x)$
(or)	$\frac{4}{10}$	$\frac{6}{25}$	$\frac{18}{225}$	$\frac{27}{125}$	**M1, A1, A1** all 4 values

1b.

$E(X) = (0 \times 0.4) + \ldots + (30 \times 0.216)$	**M1**
$E(X) = 11.76$ (awrt 11.8)	**A1**
$E(X^2) = (10^2 \times 0.24) + \ldots + (302 \times 0.216) = 276$	**A1**
Std Dev = $\sqrt{276 - 11.76^2} = 11.7346 \ldots$	**M1**
Std Dev = 11.7 3.s.f	**A1**