A-Level Starter Activity

Jethwa Maths

Topic: Laws of Logarithms

Chapter Reference: Pure 1, Chapter 14

7 minutes

1. Express lg 5 + lg 4 in the form lg *n* **(2)**

2. $\log 10^3 - \log 40$ **(2)**

3. $\log_3 54 - \log_3 2$ **(3)**

4. 2 log 2 + log 25 **(3)**

5. Express $\log_{10}\frac{1}{ab}$ in the form $p\log_{10}a + q\log_{10}b$ **(2)**

6. $\frac{1}{2}\log_5(1\frac{9}{16}) + 2\log_5 10$ **(4)**

Solutions

1.

= log (5 x 4)	**M1**
= log 20	**M1**

2.

log 10^3 – log 40 = log 1000 – log 40	**M1**
= log (1000 ÷ 40) = log 25	**M1**

3.

$\log_3(54 \div 2)$ $= \log_3 27$	**M1**
$= \log_3 3^3$	**M1**
= 3	**M1**

4.

log 2^2 + log 25 = log 4 + log 25	**M1**
= log (4 × 25) = log 100	**M1**
= log 10^2 2	**M1**

5.

$-\log_{10} ab$	**M1**
$= -\log_{10} a - \log_{10} b$	**M1**

6.

$\frac{1}{2}\log_5(\frac{25}{16}) + \log_5 10^2$ $= \log_5(\frac{25}{16})^{\frac{1}{2}} + \log_5 100$	**M1**
$= \log_5\frac{5}{4} + \log_5 100$ $= \log_5(\frac{5}{4} \times 100)$	**M1**
$= \log_5 125$ $= \log_5 5^3$	**M1**
= 3	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Logarithms Chapter Reference: Pure 1, Chapter 14	7 minutes

1. Solve $\log_5 25 = x$ **(2)**

2. Solve $\log_4 x = 1.5$ **(2)**

3. Solve $2\log_x 7 = 1$ **(2)**

4. Express $\log_q x^5$ in the form $p \log_q x$ (1)

5. Express $\frac{1}{2}\log_q x^{15}$ in the form $p \log_q x$ (1)

6. Express $\log_q \frac{1}{x}$ in the form $p \log_q x$ (2)

Solutions

1.

$5^x = 25$	**M1**
$x = 2$	**M1**

2.

$4^{1.5} = x$	**M1**
$x = 8$	**M1**

3.

$\log x\ 7 = \frac{1}{2}$	**M1**
$x^{0.5} = 7$ $x = 49$	**M1**

4.

$a = 5 \log_q x$	**M1**

5.

$\frac{15}{2} log_q x$	**M1**

6.

$= \log_q x^{-1}$	**M1**
$= -\log_q x$	**M1**

Topic: Working with Natural Logarithms Chapter Reference: Pure 1, Chapter 14	8 minutes

1. Given that $t = \ln x$, find expressions in terms of t for,

a. $\ln\sqrt{x}$ **(2)**

b. $\ln(e^2x)$ **(2)**

c. Hence, or otherwise, solve the equation, **(3)**

$$5 + \ln\sqrt{x} = \ln(e+2^x)$$

2. A bead is projected vertically upwards in a jar or liquid with a velocity of 13 ms^{-1}. Its velocity, v ms^{-1}, at time t seconds after projection, is given by,

$V = ce^{-kt} - 2$

a. Find the value of c. **(2)**

Given that the bead has a velocity of 7ms^{-1} after 5.1 seconds, find,

b. The value of k correct to 4 decimal places. **(2)**

Solutions

1a.

$\ln x^{\frac{1}{2}} = \frac{1}{2}\ln x$	**M1**
$= \frac{1}{2}t$	**M1**

1b.

$\ln e^2 + \ln x$	**M1**
$= 2 + t$	**M1**

1c.

$5 + \frac{1}{2}t = 2 + t$	**M1**
$t = \ln x = 6$	**M1**
$x = e^6$	**M1**

2a.

When $t = 0$, $v = 13$ $13 = c - 2$	**M1**
$c = 15$	**M1**

2b.

$7 = 15e^{-5.1k} - 2$ $e^{-5.1k} = \frac{3}{5}$	**M1**
$k = \frac{\ln\left(\frac{3}{5}\right)}{-5.1} = 0.1002$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Solving Exponential Functions Chapter Reference: Pure 1, Chapter 14	8 minutes

1. Find the value of x when $e^{\ln x} = 4$ **(1)**

2. Find the exact value of x, $e^{4x + 1} = 12$ **(2)**

3. Find the exact values of x, $\ln (10 - 3x) - e = 0$ **(2)**

4. Find in exact form, the solutions to the equation, $2e^{2x} + 12 = 11e^{x}$ **(3)**

5. Solve the following simultaneous equations, giving your answers to 2 decimal places.

$e^{5y} - x = 0$

$\ln x^4 = 7 - y$ **(5)**

Solutions

1.

$x = 4$	**M1**

2.

$4t + 1 = \ln 12$	**M1**
$x = \frac{1}{4}(\ln 12 - 1)$	**M1**

3.

$10 - 3y = e^e$	**M1**
$y = \frac{1}{3}(10 - e^e) = -1.72$	**M1**

4.

$2e^{2x} - 11e^x + 12 = 0$ $(2e^x - 3)(e^x - 4) = 0$	**M1**
$e^x = \frac{3}{2}$ $x = \ln \frac{3}{2}$	**M1**
$e^x = 4$ $x = \ln 4$	**M1**

5.

$e^{5y} - x = 0$ $5y = \ln x$	**M1**
$\ln x^4 = 7 - y$ $4 \ln x = 7 - y$	**M1**
$20y = 7 - y$ $y = \frac{1}{3}$	**M1**
$x = e^{\frac{5}{3}} = 5.29$	**M1**
$y = 0.33$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Solving Equations Using Logs Chapter Reference: Pure 1, Chapter 14	7 minutes

1. Solve the equation $3^x = 12$ **(2)**

2. Solve for x, $16 - 3^{4+x} = 0$ **(2)**

3. Solve the following equation, $2^{2x} + 2^x - 6 = 0$ **(3)**

4. Solve the equation, $3(16^x) - 4^{x+2} + 5 = 0$ **(3)**

5. Sketch the curves $y = 3^x$ and $y = (\frac{1}{3})^x$ on the same diagram. **(3)**

Solutions

1.

$x \log 3 = \log 12$	**M1**
$x = \log 12 \div \log 3$ $x = 2.26$	**M1**

2.

$(4 + x) \log 3 = \log 16$	**M1**
$x = (\log 16 \div \log 3) - 4$ $x = -1.48$	**M1**

3.

$(2^x + 3)(2^x - 2) = 0$	**M1**
$2^x = -3$ No solutions	**M1**
$2^x = 2$ $x = 1$	**M1**

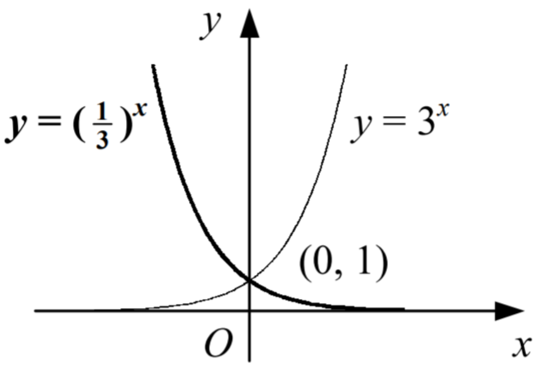
4.

$3(4^{2x}) - 16(4^x) + 5 = 0$ $(3(4^x) - 1)(4^x - 5) = 0$	**M1**
$4^x = \frac{1}{3}$ $x = \frac{\log\frac{1}{3}}{\log 4} = -0.79$	**M1**
$4^x = 5$ $x = \frac{\log 5}{\log 4} = 1.16$	**M1**

5.

Shape of graph 1 **M1**
Shape of graph 2 **M1**

Co-ordinate (0, 1) marked **M1**



A-Level Starter Activity

Jethwa Maths

Topic: Solving Equations Using Logs Chapter Reference: Pure 1, Chapter 14	**7 minutes**

1. $\log_3 x + \log_3 5 = \log_3 (2x + 3)$ **(2)**

2. $\log_5 5x - \log_5 (x + 2) = \log_5 (x + 6) - \log_5 x$ **(4)**

3. Solve the simultaneous equations:
$\log_x y = 2$
$xy = 27$ **(3)**

4. Solve the simultaneous equations:

$\log_{10} y + 2 \log_{10} x = 3$

$\log_2 y - \log_2 x = 3$ **(4)**

Solutions

1.

$\log_3 5x = \log_3 (2x + 3)$	**M1**
$5x = 2x + 3$ $x = 1$	**M1**

2.

$\log_5 \left(\frac{5x}{x+2}\right) = \log_5 \left(\frac{x+6}{x}\right)$ $\frac{5x}{x+2} = \frac{x+6}{x}$	**M1**
$5x^2 = (x + 6)(x + 2)$ $5x^2 = x^2 + 8x + 12$ $4x^2 - 8x - 12 = 0$ $x^2 - 2x - 3 = 0$	**M1**
$(x + 1)(x - 3) = 0$ $x = -1$ $x = 3$	**M1**
$\log_5 x$ is not real for $x = -1$, therefore, $x = 3$	**M1**

3.

$\log_x y = 2$ $y = x^2$	**M1**
$x^3 = 27$ $x = 3$	**M1**
$x = 3, y = 9$	**M1**

4.

$\log_{10} y + 2 \log_{10} x = 3$ $x^2 y = 10^3$	**M1**
$\log_2 y - \log_2 x = 3$ $\frac{y}{x} = 2^3$ $y = 8x$	**M1**
$8x^3 = 1000$ $x^3 = 125$ $x = 5$	**M1**
$y = 8(5) = 40$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Modelling with Logs

Chapter Reference: Pure 1, Chapter 14

7 minutes

1. A colony of fast-breeding fish is introduced into a large, newly built pond. The number of fish in the pond, n, after t weeks is modelled by,

$$n = \frac{18000}{1+8c^{-t}}$$

a. Find the initial number of fish in the pond **(2)**

Given that there are 3600 fish in the pond after 3 weeks, use this model to,

b. Show that $c = \sqrt[3]{2}$ **(4)**

c. Find the time taken for the initial population of fish to double in size, giving your answer to the nearest day. **(4)**

Solutions

1a.

$t = 0$	**M1**
$n = 2000$	**M1**

1b.

$3600 = \frac{18000}{1+8c^{-3}}$	**M1**
$1 + 8c^{-3} = 5$	**M1**
$c^{-3} = \frac{1}{2}$ $c^{3} = 2$	**M1**
$c = \sqrt[3]{2}$	**M1**

1c.

$4000 = \frac{18000}{1+8c^{-t}}$	**M1**
$1 + 8c^{-t} = \frac{9}{2}$ $c^{-t} = \frac{7}{16}$	**M1**
$-t = \frac{\log\left(\frac{7}{16}\right)}{\log \sqrt[3]{2}}$ $t = 3.758$ weeks	**M1**
$t = 25$ days	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Exponential Modelling

Chapter Reference: Pure 1, Chapter 14

5 minutes

1. A quantity N is decreasing such that at time t,

$$N = N_0 e^{kt}$$

Given that at time t = 10, N = 300 and that at time t = 20, N = 225, fin,

a. The values of the constants N0 and k (4)

b. The value of t when N = 150 (3)

Solutions

1a.

$300 = N_0 e^{10k}$ $N_0 = \frac{300}{e^{10k}}$	**M1**
$225 = \frac{300}{e^{10k}} \times e^{20k}$	**M1**
$e^{10k} = \frac{3}{4}$ $k = \frac{1}{10} \ln \frac{3}{4} = -0.0288$	**M1**
$N_0 = \frac{300}{0.75} = 400$	**M1**

1b.

$N = 400e^{-0.02877t}$	**M1**
$150 = 400e^{-0.02877t}$	**M1**
$t = \frac{-1}{0.02877} \ln \frac{3}{8} = 34.1$ (3 s.f)	**M1**