A-Level Starter Activity

Jethwa Maths

Topic: Simple Integration

Chapter Reference: Pure 1, Chapter 13

6 minutes

1. Integrate $4x^7 + 6x^{-2}$ with respect to x. **(2)**

2. Find $\int (2x + 3) dx$ **(2)**

3. Integrate $\frac{7}{\sqrt{y}}$ **(2)**

4. Find $\int y \ \ dx$ when $y = \frac{1}{4x^3} - \frac{2}{3x^2}$ **(3)**

5. Find a general expression for y given that, $\frac{dy}{dx} = (x+1)^2$ **(3)**

Solutions

1.

$4x^7 + 6x^{-2} = \frac{4x^8}{8} + \frac{6x^{-1}}{-1}$	**M1**
$= \frac{1}{2}x^8 - 6x^{-1} + c$	**M1**

2.

$\int(2x+3)dx = \frac{2x^2}{2} + 3x + \mathrm{c}$	**M1**
$= x^2 + 3x + c$	**M1**

3.

$\frac{7}{\sqrt{y}} = 7y^{-0.5}$	**M1**
$= \frac{7y^{0.5}}{0.5}$ $= 14y^{0.5} + \mathrm{c}$ or $14\sqrt{y} + c$	**M1**

4.

$y = \frac{1}{4x^3} - \frac{2}{3x^2} = \frac{1}{4}x^{-3} - \frac{2}{3}x^{-2}$	**M1**
$= \frac{1}{4\times-2}x^{-2} - \frac{2}{3\times-1}x^{-1}$ $= -\frac{1}{8}x^{-2} - \frac{2}{-3}x^{-1}$	**M1**
$= \frac{2}{3}x^{-1} - \frac{1}{8}x^{-2} + c$	**M1**

5.

$\frac{dy}{dx} = (x+1)^2 = (x+1)(x+1) = x^2 + 2x + 1$	**M1**
$y = \frac{x^3}{3} + \frac{2x^2}{2} + x$	**M1**
$y = \frac{1}{3}x^3 + x^2 + x + c$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Finding Functions

Chapter Reference: Pure 1, Chapter 13

6 minutes

1. Find an expression for y when $\frac{dy}{dx} = x^2 + 4x + 1$ and $x = -3$ and $y = 4$. **(2)**

2. The curve y = f(x) passes through the point (3, 5).
Given that $f'(x) = 3 + 2x - x^2$, find an expression for f(x). **(2)**

3. Given that $\frac{dy}{dx} = 10x^{\frac{3}{2}} - 2x^{-\frac{1}{2}}$ and $y = 7$ when $x = 0$, find the value of y when $x = 4$. **(3)**

Solutions

1.

$y = \int (x^2 + 4x + 1)dx$ $y = \frac{1}{3}x^3 + 2x^2 + x + c$	**M1**
At point (-3, 4), $4 = -9 + 18 - 3 + c$ $c = -2$	**M1**
$y = \frac{1}{3}x^3 + 2x^2 + x - 2$	

2.

$f(x) = \int (3 + 2x - x^2)dx$ $f(x) = 3x + x^2 - \frac{1}{3}x^3 + c$	**M1**
At (3, 5) $5 = 9 + 9 - 9 + c$ $c = -4$	**M1**
$f(x) = 3x + x^2 - \frac{1}{3}x^3 - 4$	

3.

$y = \int \left(10x^{\frac{3}{2}} - 2x^{-\frac{1}{2}}\right) dx$ $y = 4x^{\frac{5}{2}} - 4x^{\frac{1}{2}} + c$	**M1**
When y = 0, $x = 7$ $7 = 0 + 0 + c$ $c = 7$	**M1**
$y = 4x^{\frac{5}{2}} - 4x^{\frac{1}{2}} + 7$	
When $x = 4$, $y = 4(32) - 4(2) + 7$ $y = 127$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Definite Integrals

Chapter Reference: Pure 1, Chapter 13

8 minutes

1. Evaluate $\int_1^3 (4x - 1)dx$ **(2)**

2. Evaluate $\int_1^4 (x^3 - 2x - 7)dx$ **(3)**

3. Given that $\int_1^4 (3x^2 + ax - 5)dx = 18$, find the value of the constant *a*. **(3)**

4. Given that $\int_{-1}^k (3x^2 - 12x + 9)dx = 16$, find the value of the constant *k*. **(5)**

Solutions

1.

$[2x^2 - x]_1^3$	**M1**
$= (18 - 3) - (2 - 1)$ $= 14$	**M1**

2.

$= [\frac{1}{4}x^4 - x^2 - 7x]_1^4$	**M1**
$= (64 - 16 - 28) - (\frac{1}{4} - 1 - 7)$ $= 27.75$	**M1**

3.

$[x^3 + \frac{1}{2}ax^2 - 5x]_1^4$	**M1**
$= (64 + 8a - 20)\text{-}(1 + \frac{1}{2}a - 5)$ $= 48 + \frac{15}{2}a$	**M1**
$48 + \frac{15}{2}a = 18$ $a = -4$	**M1**

4.

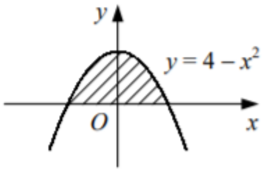
$[x^3 - \frac{1}{2}6x^2 + 9x]_{-1}^{k}$	**M1**
$= (k^3 - 6k^2 + 9k) - (-1 - 6 - 9)$ $= k^3 - 6k^2 + 9k + 16$	**M1**
$k^3 - 6k^2 + 9k + 16 = 16$ $k(k^2 - 6k + 9) = 0$ $k(k - 3)^2 = 0$	**M1**
$k = 0$ or $k = 3$	**M1**
As k cannot be 0, $k = 3$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Areas Under Graphs Chapter Reference: Pure 1, Chapter 13	10 minutes

1. The diagram shows the curve with equation $y = 4 - x^2$.



a. Find the coordinates of the points where the curves crosses the x-axis. **(2)**
b. Find the area of the shaded region enclosed by the curve and the x-axis. **(3)**

2a. Sketch the curve with the equation $y = x^2 + 4x$. **(4)**
b. Find the area of the region enclosed by the curve, the y-axis and the line $x = 2$. **(3)**

Solutions

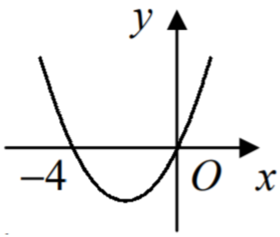
1a.

$y = 0$ $4 - x^2 = 0$ $x^2 = 4$	**M1**
$x = \pm 2$	**M1**
(-2, 0) and (2, 0)	

1b.

$\int_{-2}^{2}(4 - x^2)dx = [4x - \frac{1}{3}x^3]_{-2}^{2}$	**M1**
$= (8 - \frac{8}{3}) - (-8 + \frac{8}{3})$	**M1**
$= \frac{32}{3}$	**M1**

2a.

$x^2 + 4x = 0$ $x(x + 4) = 0$	**M1**
$x = -4$ $x = 0$	**M1**
Shape **M1** Co-ordinates (-4, 0) and (0, 0) marked. **M1** 	

2b.

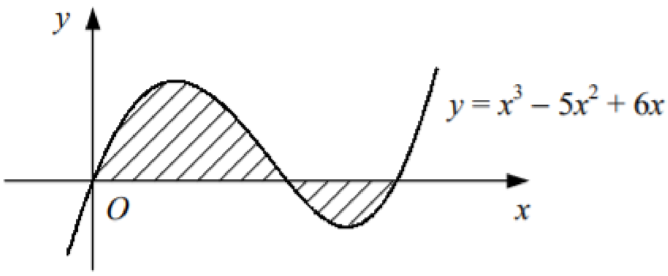
$\int_{0}^{2}(x^2 + 4x)dx = [\frac{1}{3}x^3 + 2x^2]_{0}^{2}$	**M1**
$= (\frac{8}{3} + 8) - (0)$	**M1**
$= \frac{32}{3}$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Areas Under Graphs (H) Chapter Reference: Pure 1, Chapter 13	7 minutes

1. The diagram shows the curve with equation $y = x^3 - 5x^2 + 6x$



a. Find the coordinates of the points where the curve crosses the x-axis. **(2)**

b. Show that the total area of the shaded regions enclosed by the curve and the x-axis is $3\frac{1}{12}$ **(5)**

<u>**Solutions**</u>

1a.

$x^3 - 5x^2 + 6x = 0$ $x(x-2)(x-3) = 0$	**M1**
$x = 0$, 2 and 3	**M1**
(0, 0) (2, 0) (3, 0)	

1b.

$\int_0^2 (x^3 - 5x^2 + 6x)dx$ $= [\frac{1}{4}x^4 - \frac{5}{3}x^3 + 3x^2]_0^2$	**M1**
$= (4 - \frac{40}{3} + 12) - 0 = \frac{8}{3}$	**M1**
$\int_2^3 (x^3 - 5x^2 + 6x)dx$ $= = [\frac{1}{4}x^4 - \frac{5}{3}x^3 + 3x^2]_2^3$	**M1**
$= (\frac{81}{4} - 45 + 27) - 8/3$ $= -\frac{5}{12}$	**M1**
Total Area $= \frac{8}{3} + \frac{5}{12} = 3\frac{1}{12}$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Areas Between Curves and Lines Chapter Reference: Pure 1, Chapter 13	7 minutes

1. The diagram shows the curve with equation $y = 2x^2 + 6x + 7$ and the straight line with the equation $4x - y + 11 = 0$.

a. Find the coordinates of the points where the curve and line intersect. **(4)**

b. Find the area of the shaded region enclosed by the curve and the line. **(6)**

Solutions

1a.

$4x - y + 11 = 0$ $y = 4x + 11$	**M1**
$2x^2 + 6x + 7 = 4x + 11$	**M1**
$x^2 + x - 2 = 0$ $(x + 2)(x - 1) = 0$	**M1**
$x = -2$ $x = 1$	**M1**
(-2, 3) and (1, 15)	

1b.

Area below curve, $\int_{-2}^{1}(2x^2 + 6x + 7)\, dx$	**M1**
$= [\frac{2}{3}x^3 + 3x^2 + 6x]_{-2}^{1}$	**M1**
$= (\frac{2}{3} + 3 + 7) - \left(-\frac{16}{3} + 12 - 14\right)$	**M1**
$= 18$	**M1**
Area below line: $= \frac{1}{2} \times 3 \times (3 + 15)$ $= 27$	**M1**
Area between line and curve, $= 27 - 18$ $= 9$	**M1**