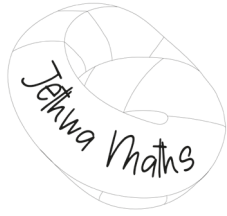


A-Level Starter Activity



Topic: Simple Integration

Chapter Reference: Pure 1, Chapter 13

**6
minutes**

1. Integrate $4x^7 + 6x^{-2}$ with respect to x .

(2)

2. Find $\int (2x + 3)dx$

(2)

3. Integrate $\frac{7}{\sqrt{y}}$

(2)

4. Find $\int y \, dx$ when $y = \frac{1}{4x^3} - \frac{2}{3x^2}$

(3)

5. Find a general expression for y given that, $\frac{dy}{dx} = (x + 1)^2$

(3)

Solutions

1.

$4x^7 + 6x^{-2} = \frac{4x^8}{8} + \frac{6x^{-1}}{-1}$	M1
$= \frac{1}{2}x^8 - 6x^{-1} + c$	M1

2.

$\int(2x + 3)dx = \frac{2x^2}{2} + 3x + c$	M1
$= x^2 + 3x + c$	M1

3.

$\frac{7}{\sqrt{y}} = 7y^{-0.5}$	M1
$= \frac{7y^{0.5}}{0.5}$	M1
$= 14y^{0.5} + c$ or $14\sqrt{y} + c$	

4.

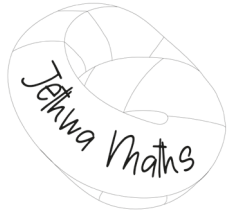
$y = \frac{1}{4x^3} - \frac{2}{3x^2} = \frac{1}{4}x^{-3} - \frac{2}{3}x^{-2}$	M1
$= \frac{1}{4 \times -2}x^{-2} - \frac{2}{3 \times -1}x^{-1}$	M1
$= -\frac{1}{8}x^{-2} - \frac{2}{-3}x^{-1}$	
$= \frac{2}{3}x^{-1} - \frac{1}{8}x^{-2} + c$	M1

5.

$\frac{dy}{dx} = (x + 1)^2 = (x + 1)(x + 1) = x^2 + 2x + 1$	M1
$y = \frac{x^3}{3} + \frac{2x^2}{2} + x$	M1
$y = \frac{1}{3}x^3 + x^2 + x + c$	M1



A-Level Starter Activity



Topic: Finding Functions
Chapter Reference: Pure 1, Chapter 13

**6
minutes**

1. Find an expression for y when $\frac{dy}{dx} = x^2 + 4x + 1$ and $x = -3$ and $y = 4$.

(2)

2. The curve $y = f(x)$ passes through the point $(3, 5)$.
Given that $f'(x) = 3 + 2x - x^2$, find an expression for $f(x)$.

(2)

3. Given that $\frac{dy}{dx} = 10x^{\frac{3}{2}} - 2x^{-\frac{1}{2}}$ and $y = 7$ when $x = 0$, find the value of y when $x = 4$.

(3)

Solutions

1.

$y = \int (x^2 + 4x + 1) dx$ $y = \frac{1}{3}x^3 + 2x^2 + x + c$	M1
At point (-3, 4), $4 = -9 + 18 - 3 + c$ $c = -2$	M1
$y = \frac{1}{3}x^3 + 2x^2 + x - 2$	

2.

$f(x) = \int (3 + 2x - x^2) dx$ $f(x) = 3x + x^2 - \frac{1}{3}x^3 + c$	M1
At (3, 5) $5 = 9 + 9 - 9 + c$ $c = -4$	M1
$f(x) = 3x + x^2 - \frac{1}{3}x^3 - 4$	

3.

$y = \int \left(10x^{\frac{3}{2}} - 2x^{-\frac{1}{2}} \right) dx$ $y = 4x^{\frac{5}{2}} - 4x^{\frac{1}{2}} + c$	M1
When $y = 0, x = 7$ $7 = 0 + 0 + c$ $c = 7$	M1
$y = 4x^{\frac{5}{2}} - 4x^{\frac{1}{2}} + 7$	
When $x = 4,$ $y = 4(32) - 4(2) + 7$ $y = 127$	M1



A-Level Starter Activity



Topic: Definite Integrals
Chapter Reference: Pure 1, Chapter 13

**8
minutes**

1. Evaluate $\int_1^3 (4x - 1)dx$

(2)

2. Evaluate $\int_1^4 (x^3 - 2x - 7)dx$

(3)

3. Given that $\int_1^4 (3x^2 + ax - 5)dx = 18$, find the value of the constant a .

(3)

4. Given that $\int_{-1}^k (3x^2 - 12x + 9)dx = 16$, find the value of the constant k .

(5)

Solutions

1.

$[2x^2 - x]_1^3$	M1
$= (18 - 3) - (2 - 1)$	M1
$= 14$	

2.

$= [\frac{1}{4}x^4 - x^2 - 7x]_1^4$	M1
$= (64 - 16 - 28) - (\frac{1}{4} - 1 - 7)$	M1
$= 27.75$	

3.

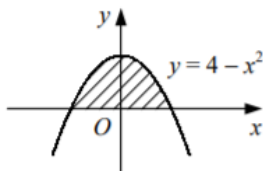
$[x^3 + \frac{1}{2}ax^2 - 5x]_1^4$	M1
$= (64 + 8a - 20) - (1 + \frac{1}{2}a - 5)$	M1
$= 48 + \frac{15}{2}a$	M1
$48 + \frac{15}{2}a = 18$	M1
$a = -4$	

4.

$[x^3 - \frac{1}{2}6x^2 + 9x]_{-1}^k$	M1
$= (k^3 - 6k^2 + 9k) - (-1 - 6 - 9)$	M1
$= k^3 - 6k^2 + 9k + 16$	M1
$k^3 - 6k^2 + 9k + 16 = 16$	
$k(k^2 - 6k + 9) = 0$	M1
$k(k - 3)^2 = 0$	
$k = 0$ or $k = 3$	M1
As k cannot be 0, $k = 3$	M1



1. The diagram shows the curve with equation $y = 4 - x^2$.



a. Find the coordinates of the points where the curves crosses the x -axis. (2)

b. Find the area of the shaded region enclosed by the curve and the x -axis. (3)

2a. Sketch the curve with the equation $y = x^2 + 4x$. (4)

b. Find the area of the region enclosed by the curve, the y -axis and the line $x = 2$. (3)

Solutions

1a.

$y = 0$ $4 - x^2 = 0$ $x^2 = 4$	M1
$x = \pm 2$	M1
$(-2, 0)$ and $(2, 0)$	

1b.

$\int_{-2}^2 (4 - x^2) dx = [4x - \frac{1}{3}x^3]_{-2}^2$	M1
$= (8 - \frac{8}{3}) - (-8 + \frac{8}{3})$	M1
$= \frac{32}{3}$	M1

2a.

$x^2 + 4x = 0$ $x(x + 4) = 0$	M1
$x = -4$ $x = 0$	M1
Shape M1 Co-ordinates $(-4, 0)$ and $(0, 0)$ marked. M1	

2b.

$\int_0^2 (x^2 + 4x) dx = [\frac{1}{3}x^3 + 2x^2]_0^2$	M1
$= (\frac{8}{3} + 8) - (0)$	M1
$= \frac{32}{3}$	M1

Solutions

1a.

$x^3 - 5x^2 + 6x = 0$ $x(x-2)(x-3) = 0$	M1
$x = 0, 2 \text{ and } 3$	M1
$(0, 0)$ $(2, 0)$ $(3, 0)$	

1b.

$\int_0^2 (x^3 - 5x^2 + 6x) dx$ $= \left[\frac{1}{4}x^4 - \frac{5}{3}x^3 + 3x^2 \right]_0^2$	M1
$= \left(4 - \frac{40}{3} + 12 \right) - 0 = \frac{8}{3}$	M1
$\int_2^3 (x^3 - 5x^2 + 6x) dx$ $= \left[\frac{1}{4}x^4 - \frac{5}{3}x^3 + 3x^2 \right]_2^3$	M1
$= \left(\frac{81}{4} - 45 + 27 \right) - \frac{8}{3}$ $= -\frac{5}{12}$	M1
Total Area $= \frac{8}{3} + \frac{5}{12} = 3\frac{1}{12}$	M1



Solutions

1a.

$4x - y + 11 = 0$ $y = 4x + 11$	M1
$2x^2 + 6x + 7 = 4x + 11$	M1
$x^2 + x - 2 = 0$ $(x + 2)(x - 1) = 0$	M1
$x = -2$ $x = 1$	M1
$(-2, 3)$ and $(1, 15)$	

1b.

Area below curve, $\int_{-2}^1 (2x^2 + 6x + 7) dx$	M1
$= \left[\frac{2}{3}x^3 + 3x^2 + 6x \right]_{-2}^1$	M1
$= \left(\frac{2}{3} + 3 + 7 \right) - \left(-\frac{16}{3} + 12 - 14 \right)$	M1
$= 18$	M1
Area below line: $= \frac{1}{2} \times 3 \times (3 + 15)$ $= 27$	M1
Area between line and curve, $= 27 - 18$ $= 9$	M1

