A-Level Starter Activity

Jethwa Maths

Topic: Differentiation Chapter Reference: Pure 1, Chapter 12	**8 minutes**

1. Find $\frac{dy}{dx}$ when $y = x^5 + x^2$ **(1)**

2. Find $\frac{dy}{dx}$ when $y = 6x^3 + 5x^{-2}$ **(1)**

3. Find $\frac{dy}{dx}$ when $y = 3x^{-1} - 5x^{-\frac{3}{2}}$ **(1)**

4. Find $\frac{dy}{dx}$ when $y = (x + 1)(x + 6)$ **(2)**

5. Find $\frac{dy}{dx}$ when $y = \sqrt{x}(x - 4)$ **(2)**

6. Find $\frac{dy}{dx}$ when $y = \frac{5+\sqrt{x}}{x^2}$ **(2)**

A-Level Starter Activity

Jethwa Maths

Topic: Differentiation Chapter Reference: Pure 1, Chapter 10	10 minutes

1. Find the value of the gradient of the curve $y = 3x^2 + x - 5$ at the point $x = 1$ **(2)**

2. Find the value of the gradient of the curve $y = (x + 1)^2$ at the point (4, 25) **(2)**

3. Find the value of the gradient of the curve $y = \frac{8x+x^3}{4\sqrt{x}}$ **(2)**

4. Find the equation of the tangent to the curve $y = 3 - x^2$ at the point (-3, -6) **(4)**

Solutions

1.

$y = 3x^2 + x - 5$ $\frac{dy}{dx} = 6x + 1$	**M1**
At $x = 1$, $\frac{dy}{dx} = 6(1) + 1 = 7$	**M1**

2.

$y = (x + 1)^2 = (x + 1)(x + 1)$ $y = x^2 + 2x + 1$	**M1**
$\frac{dy}{dx} = 2x + 2$	**M1**
When $x = 4$, $\frac{dy}{dx} = 2(4) + 2 = 10$	**M1**

3.

$y = \frac{8x+x^3}{4\sqrt{x}} = \frac{8x}{4x^{\frac{1}{2}}} + \frac{x^3}{4x^{\frac{1}{2}}} = 2x^{0.5} + \frac{1}{4}x^{2.5}$	**M1**
$\frac{dy}{dx} = x^{-0.5} + \frac{5}{8}x^{1.5}$	**M1**

4.

$y = 3 - x^2$ $\frac{dy}{dx} = -2x$	**M1**
When $x = -3$, $\frac{dy}{dx} = -2(-3) = 6$	**M1**
Equation of line crossing (-3, -6) $-6 = 6(-3) + c$ $-6 = -18 + c$ $c = 12$	**M1**
$y = 6x + 12$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Differentiating Quadratics Chapter Reference: Pure 1, Chapter 12	6 minutes

1. Find $\frac{dy}{dx}$ of the curve, $y = x^2 + 4x + 6$ **(1)**

2. Find $\frac{dy}{dx}$ of the curve, $y = 2x^2 - 5x - 2$ **(1)**

3. Find $\frac{dy}{dx}$ of the curve, $y = (x + 5)(4x - 2)$ **(2)**

4. Find $\frac{dy}{dx}$ of the curve, $y = (-9x + 3)(2x - 4)$ **(2)**

5. Find the values of x for which $\frac{dy}{dx} = 0$ when $y = (x - 3)(x + 5)$ **(4)**

Solutions

1.

$y = x^2 + 4x + 6$ $\frac{dy}{dx} = 2x + 4$	**M1**

2.

$y = 2x^2 - 5x - 2$ $\frac{dy}{dx} = 4x - 5$	**M1**

3.

$y = (x + 5)(4x - 2)$ $y = x^2 + 18x - 10$	**M1**
$\frac{dy}{dx} = 2x + 18$	**M1**

4.

$y = (-9x + 3)(2x - 4)$ $y = -18x^2 + 42x - 12$	**M1**
$\frac{dy}{dx} = -36x + 42$	**M1**

5.

$y = (x - 3)(x + 5)$ $y = x^2 + 2x - 15$	**M1**
$\frac{dy}{dx} = 2x + 2$	**M1**
$2x + 2 = 0$ $2x = -2$	**M1**
$x = -1$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Differentiation and Normals

Chapter Reference: Pure 1, Chapter 12

7 minutes

1. A curve has the equation $y = 2 + \frac{4}{x}$.

a. Find an equation of the normal to the curve at the point M (4, 3).

The normal to the curve at M intersects the curve again at the point N.

b. Find the coordinates of the point N. **(3)**

2. A curve has the equation $y = (x + 2)(x - 5)$

a. Find an equation of the normal to the curve at the point $P(2, -12)$. **(3)**

Solutions

1a.

$y = 2 + \frac{4}{x}$ $\frac{dy}{dx} = -4x^{-2}$	**M1**
Gradient at $M = -\frac{1}{4}$ Gradient of normal = 4	**M1**
$y - 3 = 4(x - 4)$ $y = 4x - 13$	**M1**

1b.

$4x - 13 = 2 + \frac{4}{x}$ $4x^2 - 15x - 4 = 0$	**M1**
$(4x + 1)(x - 4) = 0$	**M1**
$x = 4$ (at M) $x = -\frac{1}{4}$ Therefore, $N(-\frac{1}{4}, -14)$	**M1**

2.

$y = (x + 2)(x - 5)$ $y = x^2 - 3x - 10$	**M1**
$\frac{dy}{dx} = 2x - 3$	**M1**
At $x = 2$, $\frac{dy}{dx} = 2(2) - 3 = 1$ Therefore, gradient of normal = -1	**M1**
$y - -12 = -1(x - 2)$ $y + 12 = -x + 2$ $y = -x - 10$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Increasing and Decreasing Functions Chapter Reference: Pure 1, Chapter 12	8 minutes

1. Find the set of values of x for which $f(x)$ is increasing when, $f(x) = 2x^2 + 2x + 1$ **(3)**

2. Find the set of values of x for which $f(x)$ is decreasing when $f(x) = x(x - 6)^2$ **(4)**

3. $f(x) = x^3 + kx^2 + 3$

Given that $(x + 1)$ is a factor of $f(x)$

a. Find the values of the constant k, **(2)**

b. Find the set of values of x for which $f(x)$ is increasing. **(3)**

Solutions

1.

$f'(x) = 4x + 2$	**M1**
$4x + 2 > 0$	**M1**
$x > -\frac{1}{2}$	**M1**

2.

$f(x) = x(x - 6)^2$ $f(x) = x(x^2 - 12x + 36)$ $f(x) = x^3 - 12x^2 + 36x$	**M1**
$f'(x) = 3x^2 - 24x + 36$	**M1**
$3x^2 - 24x + 36 < 0$ $x^2 - 8x + 12 < 0$	**M1**
$(x - 6)(x - 2) < 0$ $2 < x < 6$	**M1**

3a.

As $(x + 1)$ is a factor, $f(-1) = 0$	**M1**
Therefore, $-1 + k + 3 = 0$ $k = -2$	**M1**

3b.

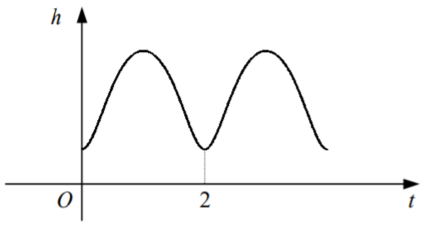
$f'(x) = 3x^2 - 4x$	**M1**
$3x^2 - 4x > 0$ $x(3x - 4) > 0$	**M1**
$x < 0$ and $x > \frac{4}{3}$	**M1**

A-Level Starter Activity

Topic: Modelling with Differentiation Chapter Reference: Pure 1, Chapter 12	5 minutes

1. The graph shows the height, h cm of letters on a website advert t seconds after the advert appears on the screen. For t in the interval $0 \leq t \leq 2$, h is given by the equation,

$$H = 2t^4 - 8t^3 + 8t^2 + 1$$



For larger values of t, the variation of h over this interval is repeated every 2 seconds.

a. Find $\frac{dh}{dt}$ for t in the interval $0 \leq t \leq 2$ **(1)**

b. Find the rate at which the maximum height of the letters is increasing when $t = 0.25$. **(1)**

c. Find the maximum height of the letters. **(3)**

Solutions

1a.

$\frac{dh}{dt} = 8t^3 - 24t^2 + 16t$	**M1**

1b.

When $t = 0.25$, $\frac{dh}{dt} = 2.625$ cm per second.	**M1**

1c.

Stationary point: $8t^3 - 24t^2 + 16t = 0$	**M1**
$8t(t - 1)(t - 2) = 0$ $t = 0$ $t = 1$ $t = 2$	**M1**
From graph, maximum occurs when $t = 1$, Therefore, maximum height is 3 cm.	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Second Order Derivatives Chapter Reference: Pure 1, Chapter 12	8 minutes

1. Find $\frac{d^2y}{dx^2}$ when $y = x^3 - 6x^2 - 36x + 15$ **(2)**

2. Find the stationary points of $y = \frac{x^4+16}{2x^2}$ and state if it is a maximum or minimum. **(6)**

Solutions

1a.

$\frac{dy}{dx} = 3x^2 - 12x - 36$	**M1**
$\frac{d^2y}{dx^2} = 6x - 12$	**M1**

1b.

$y = \frac{1}{2}x^2 + 8x^{-2}$	**M1**
$\frac{dy}{dx} = x - 16x^{-3}$ At stationary point, $\frac{dy}{dx} = 0$	**M1**
$x - 16x^{-3} = 0$ $x^4 = 16$ $x = \pm 2$	**M1**
$\frac{d^2y}{dx^2} = 1 + 48x^{-4}$	**M1**
At (-2, 4), $\frac{d^2y}{dx^2} = 1 + 48(-2)^{-4} = 4$ Therefore, minimum point	**M1**
At (2, 4), $\frac{d^2y}{dx^2} = 1 + 48(2)^{-4} = 4$ Therefore, minimum point	**M1**

Topic: Simple Differentiation

Chapter Reference: Pure 1, Chapter 12

7 minutes

1. Complete the table:

Curve	$\frac{dy}{dx}$
$y = x^5$	
$f(x) = 2x^3$	
$f(x) = x^{-3}$	
$y = 5x^{-8}$	
$y = 4x^3 + 3x^{-4}$	
$f(x) = 2x + \frac{1}{3}x^6$	
$f(x) = 7 + x^{-\frac{4}{5}}$	
$y = 3x^{-1} - 5x^{-\frac{3}{2}}$	
$f(x) = 2 - 7x^{-1} + x^{-\frac{8}{3}}$	
$y = \frac{1}{4x} - \frac{1}{x^2}$	

(10)

Solutions

Curve	$\frac{dy}{dx}$	
$y = x^5$	$\frac{dy}{dx} = 5x^4$	**M1**
$f(x) = 2x^3$	$f'(x) = 6x^2$	**M1**
$f(x) = x^{-3}$	$f'(x)\ -3x^{-4}$	**M1**
$y = 5x^{-8}$	$\frac{dy}{dx} = -40x^{-9}$	**M1**
$y = 4x^3 + 3x^{-4}$	$\frac{dy}{dx} = 12x^2 - 12x^{-5}$	**M1**
$f(x) = 2x + \frac{1}{3}x^6$	$f'(x) = 2 + 2x^5$	**M1**
$f(x) = 7 + x^{-\frac{4}{5}}$	$f'(x) = -\frac{4}{5}x^{-1.8}$	**M1**
$y = 3x^{-1} - 5x^{-\frac{3}{2}}$	$\frac{dy}{dx} = -3x^{-2} + \frac{15}{2}x^{-2.5}$	**M1**
$f(x) = 2 - 7x^{-1} + x^{-\frac{8}{3}}$	$f'(x) = 7x^{-2} - \frac{8}{3}x^{-\frac{11}{3}}$	**M1**
$y = \frac{1}{4x} - \frac{1}{x^2}$	$\frac{dy}{dx} = -4x^{-2} + 2x^{-3}$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Stationary Points Chapter Reference: Pure 1, Chapter 12	8 minutes

1. The curve $y = x^3 - 3x + 1$ is stationary at the points P and Q.

a. Find the coordinates of the points P and Q. **(4)**

b. Find the length of PQ in the form $k\sqrt{5}$ **(2)**

2a. Find the coordinates of the stationary points on the curve, $y = 2 + 9x + 3x^2 - x^3$ **(4)**

b. Determine whether each stationary point is a maximum or minimum point. **(3)**

<u>**Solutions**</u>

1a.

$\frac{dy}{dx} = 3x^2 - 3$ At stationary point, $\frac{dy}{dx} = 0$	**M1**
$3x^2 - 3 = 0$ $x^2 = 1$ $x = \pm 1$	**M1**
When $x = 1$, $y = -1$	**M1**
When $x = -1$, $y = 3$	**M1**

1b.

$PQ^2 = 2^2 + 4^2 = 20$	**M1**
$PQ = \sqrt{20} = 2\sqrt{5}$	**M1**

2a.

$\frac{dy}{dx} = 9 + 6x - 3x^2$ At stationary point, $\frac{dy}{dx} = 0$	**M1**
$9 + 6x - 3x^2 = 0$ $-3(x+1)(x - 3) = 0$ $x = -1, 3$	**M1**
When $x = -1$, $y = -3$	**M1**
When $x = 3$ $y = 29$	**M1**
(-1, -3) and (3, 29)	

2b.

$\frac{d^2y}{dx^2} = 6 - 6x$	**M1**
At (-1, -3), $\frac{d^2y}{dx^2} = 6 - 6(-1) = 12$ Therefore, minimum point	**M1**
At (3, 29), $\frac{d^2y}{dx^2} = 6 - 6(3) = -12$ Therefore, maximum point	**M1**