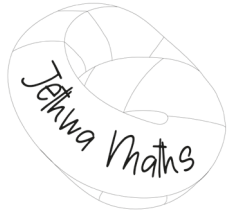


A-Level Starter Activity



Topic: Differentiation

Chapter Reference: Pure 1, Chapter 12

8

minutes

1. Find $\frac{dy}{dx}$ when $y = x^5 + x^2$ (1)

2. Find $\frac{dy}{dx}$ when $y = 6x^3 + 5x^{-2}$ (1)

3. Find $\frac{dy}{dx}$ when $y = 3x^{-1} - 5x^{-\frac{3}{2}}$ (1)

4. Find $\frac{dy}{dx}$ when $y = (x + 1)(x + 6)$ (2)

5. Find $\frac{dy}{dx}$ when $y = \sqrt{x}(x - 4)$ (2)

6. Find $\frac{dy}{dx}$ when $y = \frac{5 + \sqrt{x}}{x^2}$ (2)

Solutions

1.

$y = x^5 + x^2$ $\frac{dy}{dx} = 5x^4 + 2x$	M1
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2.

$y = 6x^3 + 5x^{-2}$ $\frac{dy}{dx} = 12x^2 - 10x^{-3}$	M1
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3.

$y = 3x^{-1} - 5x^{-\frac{3}{2}}$ $\frac{dy}{dx} = -3x^{-2} + \frac{15}{2}x^{-\frac{5}{2}}$	M1
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4.

$y = (x + 1)(x + 6)$ $y = x^2 + 7x + 6$	M1
$\frac{dy}{dx} = 2x + 7$	M1

5.

$y = \sqrt{x}(x - 4)$ $y = x^{1.5} - 4x^{0.5}$	M1
$\frac{dy}{dx} = 1.5x^{0.5} - 4x^{-0.5}$	M1

6.

$y = \frac{5 + \sqrt{x}}{x^2}$ $y = 5x^{-2} + x^{-2.5}$	M1
$\frac{dy}{dx} = -10x^{-3} - 1.5x^{-2.5}$	M1



A-Level Starter Activity



Topic: Differentiation

Chapter Reference: Pure 1, Chapter 10

**10
minutes**

1. Find the value of the gradient of the curve $y = 3x^2 + x - 5$ at the point $x = 1$ (2)

2. Find the value of the gradient of the curve $y = (x + 1)^2$ at the point $(4, 25)$ (2)

3. Find the value of the gradient of the curve $y = \frac{8x+x^3}{4\sqrt{x}}$ (2)

4. Find the equation of the tangent to the curve $y = 3 - x^2$ at the point $(-3, -6)$ (4)

Solutions

1.

$y = 3x^2 + x - 5$ $\frac{dy}{dx} = 6x + 1$	M1
At $x = 1$, $\frac{dy}{dx} = 6(1) + 1 = 7$	M1

2.

$y = (x + 1)^2 = (x + 1)(x + 1)$ $y = x^2 + 2x + 1$	M1
$\frac{dy}{dx} = 2x + 2$	M1
When $x = 4$, $\frac{dy}{dx} = 2(4) + 2 = 10$	M1

3.

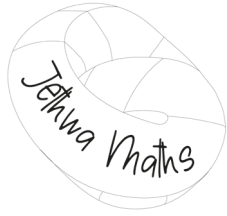
$y = \frac{8x+x^3}{4\sqrt{x}} = \frac{8x}{4x^{\frac{1}{2}}} + \frac{x^3}{4x^{\frac{1}{2}}} = 2x^{0.5} + \frac{1}{4}x^{2.5}$	M1
$\frac{dy}{dx} = x^{-0.5} + \frac{5}{8}x^{1.5}$	M1

4.

$y = 3 - x^2$ $\frac{dy}{dx} = -2x$	M1
When $x = -3$, $\frac{dy}{dx} = -2(-3) = 6$	M1
Equation of line crossing $(-3, -6)$ $-6 = 6(-3) + c$ $-6 = -18 + c$ $c = 12$	M1
$y = 6x + 12$	M1



A-Level Starter Activity



Topic: Differentiating Quadratics

Chapter Reference: Pure 1, Chapter 12

6
minutes

1. Find $\frac{dy}{dx}$ of the curve, $y = x^2 + 4x + 6$ (1)

2. Find $\frac{dy}{dx}$ of the curve, $y = 2x^2 - 5x - 2$ (1)

3. Find $\frac{dy}{dx}$ of the curve, $y = (x + 5)(4x - 2)$ (2)

4. Find $\frac{dy}{dx}$ of the curve, $y = (-9x + 3)(2x - 4)$ (2)

5. Find the values of x for which $\frac{dy}{dx} = 0$ when $y = (x - 3)(x + 5)$ (4)

Solutions

1.

$y = x^2 + 4x + 6$ $\frac{dy}{dx} = 2x + 4$	M1
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2.

$y = 2x^2 - 5x - 2$ $\frac{dy}{dx} = 4x - 5$	M1
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3.

$y = (x + 5)(4x - 2)$ $y = x^2 + 18x - 10$	M1
$\frac{dy}{dx} = 2x + 18$	M1

4.

$y = (-9x + 3)(2x - 4)$ $y = -18x^2 + 42x - 12$	M1
$\frac{dy}{dx} = -36x + 42$	M1

5.

$y = (x - 3)(x + 5)$ $y = x^2 + 2x - 15$	M1
$\frac{dy}{dx} = 2x + 2$	M1
$2x + 2 = 0$	M1
$2x = -2$	M1
$x = -1$	M1



Solutions

1a.

$y = 2 + \frac{4}{x}$ $\frac{dy}{dx} = -4x^{-2}$	M1
Gradient at $M = -\frac{1}{4}$ Gradient of normal = 4	M1
$y - 3 = 4(x - 4)$ $y = 4x - 13$	M1

1b.

$4x - 13 = 2 + \frac{4}{x}$ $4x^2 - 15x - 4 = 0$	M1
$(4x + 1)(x - 4) = 0$	M1
$x = 4$ (at M) $x = -\frac{1}{4}$ Therefore, $N(-\frac{1}{4}, -14)$	M1

2.

$y = (x + 2)(x - 5)$ $y = x^2 - 3x - 10$	M1
$\frac{dy}{dx} = 2x - 3$	M1
At $x = 2$, $\frac{dy}{dx} = 2(2) - 3 = 1$ Therefore, gradient of normal = -1	M1
$y - -12 = -1(x - 2)$ $y + 12 = -x + 2$ $y = -x - 10$	M1

Solutions

1a.

$\frac{dy}{dx} = -24x^{-3}$	M1
At $A, y = 3$, gradient = -3	M1
$y - 3 = -3(x - 2)$ $3x + y - 9 = 0$	M1

1b.

Tangent: $x = -1$, $-3 + y - 0 = 0$ $y = 12$	M1
For the curve, $x = -1, y = \frac{12}{1} = 12$	M1
Therefore, tangent intersects curve at $(-1, 12)$	M1

2.

$\frac{dy}{dx} = 1 - \frac{3}{2}x^{-\frac{1}{2}}$ Gradient at $P = \frac{1}{4}$	M1
$y - 1 = \frac{1}{4}(x - 4)$	M1
$y = \frac{1}{4}x$ which passes through $(0,0)$	M1

Solutions

1.

$f'(x) = 4x + 2$	M1
$4x + 2 \geq 0$	M1
$x \geq -\frac{1}{2}$	M1

2.

$f(x) = x(x - 6)^2$ $f(x) = x(x^2 - 12x + 36)$ $f(x) = x^3 - 12x^2 + 36x$	M1
$f'(x) = 3x^2 - 24x + 36$	M1
$3x^2 - 24x + 36 \leq 0$ $x^2 - 8x + 12 \leq 0$	M1
$(x - 6)(x - 2) \leq 0$ $2 \leq x \leq 6$	M1

3a.

As $(x + 1)$ is a factor, $f(-1) = 0$	M1
Therefore, $-1 + k + 3 = 0$ $k = -2$	M1

3b.

$f'(x) = 3x^2 - 4x$	M1
$3x^2 - 4x \geq 0$ $x(3x - 4) \geq 0$	M1
$x \leq 0$ and $x \geq \frac{4}{3}$	M1



Solutions

1a.

$\frac{dh}{dt} = 8t^3 - 24t^2 + 16t$	M1
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1b.

When $t = 0.25$, $\frac{dh}{dt} = 2.625$ cm per second.	M1
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1c.

Stationary point: $8t^3 - 24t^2 + 16t = 0$	M1
$8t(t - 1)(t - 2) = 0$ $t = 0$ $t = 1$ $t = 2$	M1
From graph, maximum occurs when $t = 1$, Therefore, maximum height is 3 cm.	M1



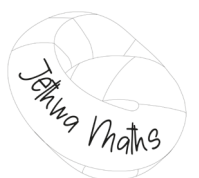
Solutions

1a.

$\frac{dy}{dx} = 3x^2 - 12x - 36$	M1
$\frac{d^2y}{dx^2} = 6x - 12$	M1

1b.

$y = \frac{1}{2}x^2 + 8x^{-2}$	M1
$\frac{dy}{dx} = x - 16x^{-3}$ At stationary point, $\frac{dy}{dx} = 0$	M1
$x - 16x^{-3} = 0$ $x^4 = 16$ $x = \pm 2$	M1
$\frac{d^2y}{dx^2} = 1 + 48x^{-4}$	M1
At $(-2, 4)$, $\frac{d^2y}{dx^2} = 1 + 48(-2)^{-4} = 4$ Therefore, minimum point	M1
At $(2, 4)$, $\frac{d^2y}{dx^2} = 1 + 48(2)^{-4} = 4$ Therefore, minimum point	M1





1. Complete the table:

Curve	$\frac{dy}{dx}$
$y = x^5$	
$f(x) = 2x^3$	
$f(x) = x^{-3}$	
$y = 5x^{-8}$	
$y = 4x^3 + 3x^{-4}$	
$f(x) = 2x + \frac{1}{3}x^6$	
$f(x) = 7 + x^{-\frac{4}{5}}$	
$y = 3x^{-1} - 5x^{-\frac{3}{2}}$	
$f(x) = 2 - 7x^{-1} + x^{-\frac{8}{3}}$	
$y = \frac{1}{4x} - \frac{1}{x^2}$	

Solutions

Curve	$\frac{dy}{dx}$	
$y = x^5$	$\frac{dy}{dx} = 5x^4$	M1
$f(x) = 2x^3$	$f'(x) = 6x^2$	M1
$f(x) = x^{-3}$	$f'(x) = -3x^{-4}$	M1
$y = 5x^{-8}$	$\frac{dy}{dx} = -40x^{-9}$	M1
$y = 4x^3 + 3x^{-4}$	$\frac{dy}{dx} = 12x^2 - 12x^{-5}$	M1
$f(x) = 2x + \frac{1}{3}x^6$	$f'(x) = 2 + 2x^5$	M1
$f(x) = 7 + x^{-\frac{4}{5}}$	$f'(x) = -\frac{4}{5}x^{-1.8}$	M1
$y = 3x^{-1} - 5x^{-\frac{3}{2}}$	$\frac{dy}{dx} = -3x^{-2} + \frac{15}{2}x^{-2.5}$	M1
$f(x) = 2 - 7x^{-1} + x^{-\frac{8}{3}}$	$f'(x) = 7x^{-2} - \frac{8}{3}x^{-\frac{11}{3}}$	M1
$y = \frac{1}{4x} - \frac{1}{x^2}$	$\frac{dy}{dx} = -4x^{-2} + 2x^{-3}$	M1

Solutions

1a.

$\frac{dy}{dx} = 3x^2 - 3$ At stationary point, $\frac{dy}{dx} = 0$	M1
$3x^2 - 3 = 0$ $x^2 = 1$ $x = \pm 1$	M1
When $x = 1$, $y = -1$	M1
When $x = -1$, $y = 3$	M1

1b.

$PQ^2 = 2^2 + 4^2 = 20$	M1
$PQ = \sqrt{20} = 2\sqrt{5}$	M1

2a.

$\frac{dy}{dx} = 9 + 6x - 3x^2$ At stationary point, $\frac{dy}{dx} = 0$	M1
$9 + 6x - 3x^2 = 0$ $-3(x+1)(x-3) = 0$ $x = -1, 3$	M1
When $x = -1$, $y = -3$	M1
When $x = 3$ $y = 29$	M1
$(-1, -3)$ and $(3, 29)$	

2b.

$\frac{d^2y}{dx^2} = 6 - 6x$	M1
At $(-1, -3)$, $\frac{d^2y}{dx^2} = 6 - 6(-1) = 12$ Therefore, minimum point	M1
At $(3, 29)$, $\frac{d^2y}{dx^2} = 6 - 6(3) = -12$ Therefore, maximum point	M1

