A-Level Starter Activity

Jotwa Maths

Topic: Magnitude and Direction of Vectors

Chapter Reference: Pure 1, Chapter 11

7 minutes

1. Find a vector of magnitude 26 in the direction $5\mathbf{i} + 12\mathbf{j}$. **(2)**

2. Find a unit vector in the direction $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ **(2)**

3. Given that $m = 2\mathbf{i} + \gamma\mathbf{j}$ and $n = \mu\mathbf{i} - 5\mathbf{j}$, find the values of γ and μ such that $m + n = 3\mathbf{i} - \mathbf{j}$ **(3)**

4. Given that $r = 6\mathbf{i} + c\mathbf{j}$, where c is a positive constant, find the value of c such that, $|r| = 10$ **(3)**

Solutions

1.

$\|5\mathrm{i} + 12\mathrm{j}\| = \sqrt{25 + 144} = 13$	**M1**
$\frac{26}{13}(5\boldsymbol{i} + 12\boldsymbol{j}) = 10\boldsymbol{i} + 24\boldsymbol{j}$	**M1**

2.

$\left\|\begin{pmatrix}4\\3\end{pmatrix}\right\| = \sqrt{16 + 9} = 5$	**M1**
$\frac{1}{5}\begin{pmatrix}4\\3\end{pmatrix}$	**M1**

3.

$(2\mathbf{i} + \gamma\boldsymbol{j}) + (\mu\mathbf{i} - 5\mathbf{j}) = 3\mathbf{i} - \mathbf{j}$	**M1**
$\gamma - 5 = -1$ $\gamma = 4$	**M1**
$2 + \mu = 3$ $\mu = 1$	**M1**

4.

$36 + c^2 = 10^2 = 100$	**M1**
$c^2 = 64$	**M1**
$c > 0$, therefore $c = 8$	**M1**

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Topic: Vectors

Chapter Reference: Pure 1, Chapter 11

7 minutes

1. The points *O, A, B* and *C* are such that $\overrightarrow{OA} = 6\boldsymbol{u} - 4\boldsymbol{v}$, $\overrightarrow{OB} = 3\boldsymbol{u} - \boldsymbol{v}$ and $\overrightarrow{OC} = \boldsymbol{v} - 3\boldsymbol{u}$, where $\boldsymbol{u}$ and $\boldsymbol{v}$ are non-parallel vectors.
The point *M* is the mid-point of *OA* and the point *N* is the point on *AB* such that *AN:NB* = 1:2.
a. Find $\overrightarrow{OM}$ and $\overrightarrow{ON}$ **(3)**
b. Prove that *C*, *M* and *N* are collinear. **(3)**

2. The points *O, A, B* and *C* are such that $\overrightarrow{OA} = 4\boldsymbol{m}$, $\overrightarrow{OB} = 4\boldsymbol{m} + 2\boldsymbol{n}$ and $\overrightarrow{OC} = 2\mathbf{m} + 3\boldsymbol{n}$, where $\boldsymbol{m}$ and $\boldsymbol{n}$ are non-parallel vectors.
a. Find an expression for $\overrightarrow{BC}$ in terms of $\boldsymbol{m}$ and $\boldsymbol{n}$. **(2)**
The point *M* is the mid-point of *OC*.
b. Show that *AM* is parallel to *BC*. **(4)**

Solutions

1a.

$\overrightarrow{OM} = \frac{1}{2}\overrightarrow{OA} = 3\boldsymbol{u} - 2\boldsymbol{v}$	**M1**
$\overrightarrow{AB} = (3\boldsymbol{u} - \boldsymbol{v}) - (6\boldsymbol{u} - 4\boldsymbol{v}) = 3\boldsymbol{v} - 3\boldsymbol{u}$	**M1**
$\overrightarrow{ON} = \overrightarrow{OA} + \frac{1}{3}\overrightarrow{AB}$ $= (6\boldsymbol{u} - 4\boldsymbol{v}) + \frac{1}{3}(3\boldsymbol{v} - 3\boldsymbol{u})$ $= 5\boldsymbol{u} - 3\boldsymbol{v}$	**M1**

1b.

$\overrightarrow{CM} = (3\boldsymbol{u} - 2\boldsymbol{v}) - (\boldsymbol{v} - 3\boldsymbol{u}) = 6\boldsymbol{u} - 3\boldsymbol{v}$	**M1**
$\overrightarrow{CN} = (5\boldsymbol{u} - 3\boldsymbol{v}) - (\boldsymbol{v} - 3\boldsymbol{u}) = 8\boldsymbol{u} - 4\boldsymbol{v}$	**M1**
$\overrightarrow{CN} = \frac{4}{3}\overrightarrow{CM}$ Therefore, $\overrightarrow{CN}$ and $\overrightarrow{CM}$ are parallel and have a common point *C*. Therefore, *C, M, N* are collinear.	**M1**

2a.

$(2\boldsymbol{m} + 3\boldsymbol{n}) - (4\boldsymbol{m} + 2\boldsymbol{n})$	**M1**
$= \boldsymbol{n} - 2\boldsymbol{m}$	**M1**

2b.

$\overrightarrow{OM} = \frac{1}{2}\overrightarrow{OC} = \boldsymbol{m} + \frac{3}{2}\boldsymbol{n}$	**M1**
$\overrightarrow{AM} = \left(\boldsymbol{m} + \frac{3}{2}\boldsymbol{n}\right) - 4\boldsymbol{m} = \frac{3}{2}\boldsymbol{n} - 3\boldsymbol{m}$	**M1**
Therefore, $\overrightarrow{AM} = \frac{3}{2}\overrightarrow{BC}$	**M1**
AM is parallel to *BC*.	**M1**

A-Level Starter Activity

Jotnwa Maths

Topic: Position Vectors

Chapter Reference: Pure 1, Chapter 11

5 minutes

1. Relative to a fixed origin O, the points A and B have position vectors $\begin{pmatrix}3\\6\end{pmatrix}$ and $\begin{pmatrix}-5\\2\end{pmatrix}$ respectively. Find,

a. The vector $\overrightarrow{AB}$ **(1)**

b. $\left|\overrightarrow{AB}\right|$ **(2)**

c. The position vector of the mid-point of AB **(2)**

d. The position vector of the point C such that $OABC$ is a parallelogram **(1)**

Solutions

1a.

$\overrightarrow{AB} = \begin{pmatrix}-5\\2\end{pmatrix} - \begin{pmatrix}3\\6\end{pmatrix} = \begin{pmatrix}-8\\-4\end{pmatrix}$	**M1**

1b.

$\left\|\overrightarrow{AB}\right\| = \sqrt{64+16}$	**M1**
$= \sqrt{80}$ $= 4\sqrt{5}$	**M1**

1c.

$\overrightarrow{OC} + \frac{1}{2}\overrightarrow{AB}$ $= \begin{pmatrix}3\\6\end{pmatrix} + \frac{1}{2}\begin{pmatrix}-8\\-4\end{pmatrix}$	**M1**
$= \begin{pmatrix}-1\\4\end{pmatrix}$	**M1**

1d.

$\overrightarrow{OC} = \overrightarrow{AB}$ Position Vector = $\begin{pmatrix}-8\\-4\end{pmatrix}$	**M1**