



1. A curve has equation $y = x^2 - 4x + 4$ and a line has equation $y = mx$, where m is a constant.

a. For the case where $m = 1$, the curve and the line intersect at the points A and B . Find the coordinates of the mid-point of AB . (4)

b. Find the non-zero value of m for which the line is a tangent to the curve, and find the coordinates of the point where the tangent touches the curve. (5)

Solutions

1a.

$y = x^2 - 4x + 4$ $y = x$ $x = x^2 - 4x + 4$	M1
$x^2 - 5x + 4 = 0$ $(x - 4)(x - 1) = 0$	M1
$x = 4, x = 1$ $y = 4, y = 1$	M1
Therefore, midpoint AB: $(\frac{1+4}{2}, \frac{1+4}{2})$ $= (\frac{5}{2}, \frac{5}{2})$	M1

1b.

$y = x^2 - 4x + 4$ $y = mx$ $mx = x^2 - 4x + 4$	M1
$x^2 - (4 + m)x + 4 = 0$	M1
As $b^2 - 4ac = 0$ $[-(4 + m)]^2 - (4)(1)(4) = 0$	M1
$(4 + m)^2 = 16$ $4 + m = \pm 4$ $m = -4 \pm 4$ Therefore, $m = 0, m = -8$	M1
When $m = -8$ $x^2 + 4x + 4 = 0$ $x = -\frac{4}{2} = -2$ Therefore $y = 16$	M1

