A-Level Starter Activity

Jethwa Maths

| Topic: Vectors<br>Chapter Reference: Pure 1, Chapter 11 | 7 minutes |
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1. The points $O$, $A$, $B$ and $C$ are such that $\overrightarrow{OA} = 6\boldsymbol{u} - 4\boldsymbol{v}$, $\overrightarrow{OB} = 3\boldsymbol{u} - \boldsymbol{v}$ and $\overrightarrow{OC} = \boldsymbol{v} - 3\boldsymbol{u}$, where $\boldsymbol{u}$ and $\boldsymbol{v}$ are non-parallel vectors.
The point $M$ is the mid-point of $OA$ and the point $N$ is the point on $AB$ such that $AN:NB = 1:2$.
a. Find $\overrightarrow{OM}$ and $\overrightarrow{ON}$ **(3)**
b. Prove that $C$, $M$ and $N$ are collinear. **(3)**

2. The points $O$, $A$, $B$ and $C$ are such that $\overrightarrow{OA} = 4\boldsymbol{m}$, $\overrightarrow{OB} = 4\boldsymbol{m} + 2\boldsymbol{n}$ and $\overrightarrow{OC} = 2\mathbf{m} + 3\boldsymbol{n}$, where $\boldsymbol{m}$ and $\boldsymbol{n}$ are non-parallel vectors.
a. Find an expression for $\overrightarrow{BC}$ in terms of $\boldsymbol{m}$ and $\boldsymbol{n}$. **(2)**
The point $M$ is the mid-point of $OC$.
b. Show that $AM$ is parallel to $BC$. **(4)**

# Solutions

1a.

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|---|---|
| $\overrightarrow{OM} = \frac{1}{2}\overrightarrow{OA} = 3\boldsymbol{u} - 2\boldsymbol{v}$ | **M1** |
| $\overrightarrow{AB} = (3\boldsymbol{u} - \boldsymbol{v}) - (6\boldsymbol{u} - 4\boldsymbol{v}) = 3\boldsymbol{v} - 3\boldsymbol{u}$ | **M1** |
| $\overrightarrow{ON} = \overrightarrow{OA} + \frac{1}{3}\overrightarrow{AB}$<br>$= (6\boldsymbol{u} - 4\boldsymbol{v}) + \frac{1}{3}(3\boldsymbol{v} - 3\boldsymbol{u})$<br>$= 5\boldsymbol{u} - 3\boldsymbol{v}$ | **M1** |

1b.

| | |
|---|---|
| $\overrightarrow{CM} = (3\boldsymbol{u} - 2\boldsymbol{v}) - (\boldsymbol{v} - 3\boldsymbol{u}) = 6\boldsymbol{u} - 3\boldsymbol{v}$ | **M1** |
| $\overrightarrow{CN} = (5\boldsymbol{u} - 3\boldsymbol{v}) - (\boldsymbol{v} - 3\boldsymbol{u}) = 8\boldsymbol{u} - 4\boldsymbol{v}$ | **M1** |
| $\overrightarrow{CN} = \frac{4}{3}\overrightarrow{CM}$<br>Therefore, $\overrightarrow{CN}$ and $\overrightarrow{CM}$ are parallel and have a common point *C*.<br>Therefore, *C, M, N* are collinear. | **M1** |

2a.

| | |
|---|---|
| $(2\boldsymbol{m} + 3\boldsymbol{n}) - (4\boldsymbol{m} + 2\boldsymbol{n})$ | **M1** |
| $= \boldsymbol{n} - 2\boldsymbol{m}$ | **M1** |

2b.

| | |
|---|---|
| $\overrightarrow{OM} = \frac{1}{2}\overrightarrow{OC} = \boldsymbol{m} + \frac{3}{2}\boldsymbol{n}$ | **M1** |
| $\overrightarrow{AM} = \left(\boldsymbol{m} + \frac{3}{2}\boldsymbol{n}\right) - 4\boldsymbol{m} = \frac{3}{2}\boldsymbol{n} - 3\boldsymbol{m}$ | **M1** |
| Therefore, $\overrightarrow{AM} = \frac{3}{2}\overrightarrow{BC}$ | **M1** |
| *AM* is parallel to *BC*. | **M1** |