



1. The curve  $y = x^3 - 3x + 1$  is stationary at the points  $P$  and  $Q$ .

a. Find the coordinates of the points  $P$  and  $Q$ .

(4)

b. Find the length of  $PQ$  in the form  $k\sqrt{5}$

(2)

2a. Find the coordinates of the stationary points on the curve,  $y = 2 + 9x + 3x^2 - x^3$

(4)

b. Determine whether each stationary point is a maximum or minimum point.

(3)

### Solutions

1a.

$\frac{dy}{dx} = 3x^2 - 3$ At stationary point, $\frac{dy}{dx} = 0$	<b>M1</b>
$3x^2 - 3 = 0$ $x^2 = 1$ $x = \pm 1$	<b>M1</b>
When $x = 1$ , $y = -1$	<b>M1</b>
When $x = -1$ , $y = 3$	<b>M1</b>

1b.

$PQ^2 = 2^2 + 4^2 = 20$	<b>M1</b>
$PQ = \sqrt{20} = 2\sqrt{5}$	<b>M1</b>

2a.

$\frac{dy}{dx} = 9 + 6x - 3x^2$ At stationary point, $\frac{dy}{dx} = 0$	<b>M1</b>
$9 + 6x - 3x^2 = 0$ $-3(x+1)(x-3) = 0$ $x = -1, 3$	<b>M1</b>
When $x = -1$ , $y = -3$	<b>M1</b>
When $x = 3$ $y = 29$	<b>M1</b>
$(-1, -3)$ and $(3, 29)$	

2b.

$\frac{d^2y}{dx^2} = 6 - 6x$	<b>M1</b>
At $(-1, -3)$ , $\frac{d^2y}{dx^2} = 6 - 6(-1) = 12$ Therefore, minimum point	<b>M1</b>
At $(3, 29)$ , $\frac{d^2y}{dx^2} = 6 - 6(3) = -12$ Therefore, maximum point	<b>M1</b>

