A-Level Unit Test: Algebra and Functions

Partial Fractions

1. Express $\frac{2x-1}{(x-1)(2x-3)}$ in partial fractions **(3)**

2. Express $\frac{2(x+5)}{8x^2+10x-3}$ in partial fractions **(3)**

3. Express $\frac{x+5}{(x+1)(x-3)^2}$ in partial fractions **(4)**

4. Express $\frac{25}{x^2(2x+1)}$ in partial fractions **(4)**

5. Given that

$$\frac{3x^4-2x^3-5x^2-4}{x^2-4} \equiv ax^2+bx+c+\frac{dx+e}{x^2-4} \qquad x \neq \pm 2$$

find the values of the constants a, b, c, d and e. **(3)**

6. Express $\frac{6x^2-x+1}{(3x-1)(x+1)} = A + \frac{B}{3x-1} + \frac{C}{x+1}$ **(4)**

7. Given that f(x) = $\frac{x^3+5x^2-2x-19}{x^2+7x+10}$.
Show that f(x) can be written in the form f(x) $\equiv x + A + \frac{B}{x+2} + \frac{C}{x+5}$, where A, B and C are integers to be found. **(5)**

Total marks: 26

Mark Scheme

1.

$\frac{2x-1}{(x-1)(2x-3)} \equiv \frac{A}{(x-1)} + \frac{B}{(2x-3)}$ $2x - 1 = A(2x - 3) + B(x - 1)$	**M1**
$x = 1.5$ $2 = A(0) + B(0.5)$ $B = 4$	**M1**
$x = 1$ $1 = A(-1) + B(0)$ $A = 1$	**M1**
$\frac{2x-1}{(x-1)(2x-3)} \equiv \frac{-1}{(x-1)} + \frac{4}{(2x-3)}$	

2.

$\frac{2(x+5)}{8x^2+10x-3} \equiv \frac{2(x+5)}{(4x-1)(2x+3)} \equiv \frac{A}{(4x-1)} + \frac{B}{(2x+3)}$ $2(x + 5) = A(2x + 3) + B(4x - 1)$	**M1**
$x = ¼$ $10.5 = 3.5A + B(0)$ $A = 3$	**M1**
$x = -1.5$ $7 = A(0) + B(-7)$ $B = 1$	**M1**
$\frac{2(x+5)}{8x^2+10x-3} \equiv \frac{3}{(4x-1)} + \frac{1}{(2x+3)}$	

3.

$\frac{x+5}{(x+1)(x-3)^2} \equiv \frac{A}{(x+1)} + \frac{B}{(x-3)} + \frac{C}{(x-3)^2}$ $x + 5 = A(x - 3)^2 + B(x + 1)(x - 3) + C(x + 1)$	**M1**
$x = 3$ $8 = A(0) + B(0) + C(4)$ $C = 2$	**M1**
$x = -1$ $4 = A(4) + B(0) + C(0)$ $A = 1$	**M1**
$x = 0$, $A = 1$, $C = 2$ $5 = 4 + B(-3) +$ $1 = -3(B)$ $B = 1$	**M1**
$\frac{x+5}{(x+1)(x-3)^2} \equiv \frac{1}{(x+1)} + \frac{1}{(x-3)} + \frac{2}{(x-3)^2}$	

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4.

$\frac{25}{x^2(2x+1)} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{2x+1}$ $25 = A(x)(2x + 1) + B(2x + 1) + C(x^2)$	**M1**
Let $x = 0$ $25 = A(0) + B(1) + C(0)$ $B = 25$	**M1**
Let $x = -\frac{1}{2}$ $25 = A(0) + B(0) + C(0.25)$ $C = 100$	**M1**
Compare x^2 coefficients: $0 = 2A + C$ $0 = 2A + 100$ $A = -50$	**M1**
$\frac{25}{x^2(2x+1)} \equiv \frac{-50}{x} + \frac{25}{x^2} + \frac{100}{2x+1}$	

5.

Long division: $3x^2 - 2x + 7$ $x^2(+0x) - 4 \overline{)3x^4 - 2x^3 - 5x^2 + (0x) - 4}$ $\underline{3x^4 + 0x^3 - 12x^2}$ $-2x^3 + 7x^2 + 0x$ $\underline{-2x^3 + 0x^2 + 8x}$ $7x^2 - 8x - 4$ $\underline{7x^2 + 0x - 28}$ $-8x + 24$	**M1** **M1**
A = 3, B = -2, C = 7, D = -8, E = 24	**M1 M1**

6.

$\frac{6x^2-x+1}{(3x-1)(x+1)} \equiv A + \frac{B}{3x-1} + \frac{C}{x+1}$ $6x^2 - x + 1 = A(3x - 1)(x + 1) + B(x + 1) + C(3x - 1)$	**M1**
Compare coefficients of x^2: $6 = 2A$, $A = 3$	**M1**
Let $x = -1$ $8 = A(0) + B(0) + C(-4)$ $C = -2$	**M1**
Let $x = \frac{1}{3}$ $\frac{4}{3} = A(0) + B\left(\frac{4}{3}\right) + C(0)$ $B = 1$	**M1**
$\frac{6x^2-x+1}{(3x-1)(x+1)} \equiv 2 + \frac{1}{3x-1} - \frac{2}{x+1}$	

7.

Long division with $\frac{x^3+5x^2-2x-19}{x^2+7x+10}$ $$\begin{array}{r} x - 2 \\ x^2+7x+10 \overline{\big) x^3 + 5x^2 - 2x - 19} \\ \underline{x^3 + 7x^2 + 10x} \\ -2x^2 - 12x - 19 \\ \underline{-2x^2 - 14x - 20} \\ 2x + 1 \end{array}$$	**M1** **A1**
$f(x) = x - 2 + \frac{2x+1}{x^2+7x+10}$ $\frac{2x+1}{x^2+7x+10} \equiv \frac{2x+1}{(x+5)(x+2)} \equiv \frac{A}{(x+5)} + \frac{B}{(x+2)}$ $2x + 1 = A(x + 2) + B(x + 5)$	**M1**
$x = -2$ $-3 = A(0) + B(3)$ $B = -1$	**M1**
$x = -5$ $-9 = A(-3) + B(0)$ $A = 3$	**M1**
$f(x) \equiv \frac{x^3+5x^2-2x-19}{x^2+7x+10} \equiv x - 2 + \frac{3}{(x+5)} - \frac{1}{(x+2)}$ $A = -2, B = -1, C = 3$	