A-Level Unit Test: Algebra and Functions

Modulus Functions

1. Given that

$$f(x) = \ln x, \quad x > 0$$

sketch on separate axes the graphs of,

a. $y = f(x)$ **(2)**

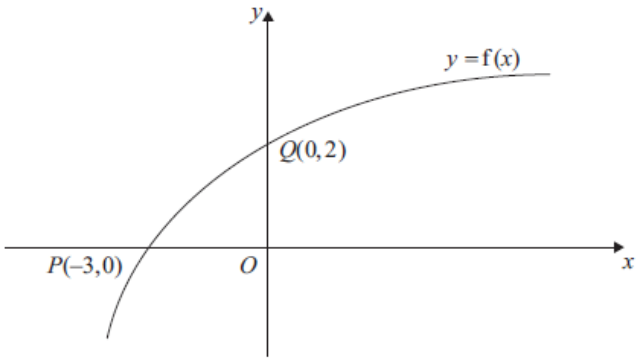
b. $y = |f(x)|$ **(2)**

c. $y = -f(x - 4)$ **(2)**

Show, on each diagram, the point where the graph meets or crosses the x-axis. In each case, state the equation of the asymptote.

2. Figure 1 shows part of the curve with equation $y = f(x)$, $x \in \mathbb{R}$

Figure 1



The curve passes through the points Q(0, 2) and P(−3, 0) as shown.

a. Find the value of ff (−3) **(2)**

On separate diagrams, sketch the curve with equation

b. $y = f^{-1}(x)$ **(2)**

c. $y = f(|x|) - 2$ **(2)**

d. $y = 2f(\frac{1}{2}x)$ **(3)**

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

3. Find the complete set of values of x for which

a. $|4x - 3| > 2 - 2x$ **(4)**

b. $|4x - 3| > \frac{3}{2} - 2x$ **(2)**

4. Given that the equation

$$|2x - a| + b = \frac{3}{2}x + 8$$

has a solution at $x = 0$ and a solution at $x = c$, find c in terms of a. **(4)**

5. Find the complete set of values of x which satisfy,

a. $|2x - 5| > 7$ **(2)**

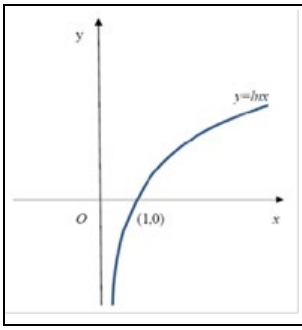
b. $|2x - 5| > x - \frac{5}{2}$. Give your answer in set notation. **(3)**

6a. Given that $|x| = 3$, find the possible values of $|2x - 1|$ **(3)**

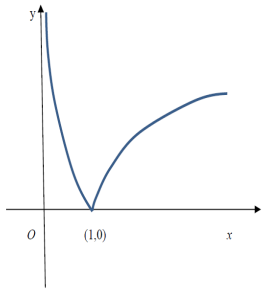
b. Solve the inequality $|x - \sqrt{2}| > |x + 3\sqrt{2}|$ **(4)**

Mark Scheme

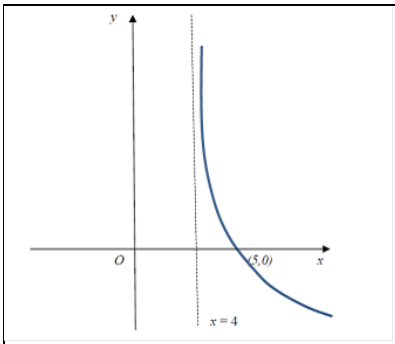
1a

	*ln* graph crossing *x* axis at (1,0) and asymptote at $x = 0$	**B1**

1b.

	Shape including cusp Touches or crosses the *x* axis at (1,0) Asymptote given as x=0	**B1ft** **B1ft** **B1**

1c.

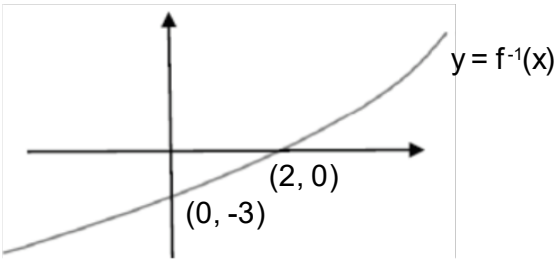
	Shape Crosses at (5, 0) Asymptote given as $x = 4$	**B1** **B1ft** **B1**

2a.

ff(-3) = f(0) = 2	**M1 A1**

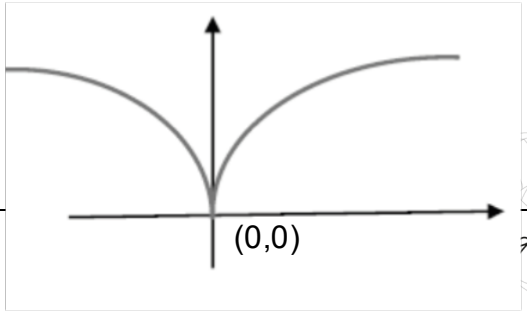
2b.

Shape **M1**
(0, -3) and (2, 0) labelled **A1**

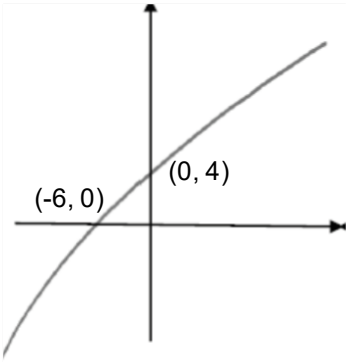


2c.

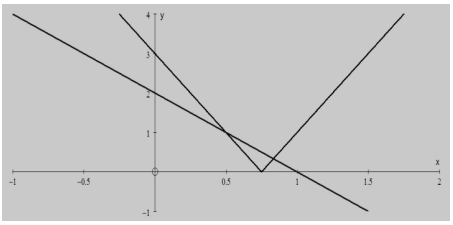
Shape **M1**
(0, 0) labelled **A1**



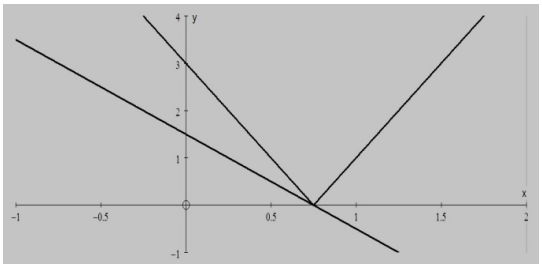
2d.

Shape **M1** (-6, 0) and (0, 4) labelled **A1** 

3a.

Solves $4x-3=2-2x$ or $3-4x=2-2x$ to give either value of x Both $x=\frac{5}{6}$ and $x=\frac{1}{2}$ or $x>\frac{5}{6}$ or $x<\frac{1}{2}$ 	**M1** **M1**
$x<\frac{1}{2}$ or $x>\frac{5}{6}$	**A1**

3b.

Draws graph Or solves $\lvert 4x-3\rvert = 1\frac{1}{2}-2x$ to give one solution x = ¾ 	**M1** **M1**
Accept for all values of x except $x=\frac{3}{4}$ Or $(x\in\square,)$ $x\neq\frac{3}{4}$, or $x<\frac{3}{4}$, $x>\frac{3}{4}$	**A1**

4.

States or uses $a+b=8$	**B1**
Attempts to solve $\lvert 2x-a\rvert+b=\frac{3}{2}x+8$ in either x or with $x=c$	
$2c-a+b=\frac{3}{2}c+8 \Rightarrow kc=\mathrm{f}(a,b)$	**M1**
Combines $kc=\mathrm{f}(a,b)$ with $a+b=8$ $\Rightarrow c=4a$	**dM1 A1**

5a.

$2x - 5 > 7$ $2x > 12$ $x > 6$	**B1**
$-(2x - 5) > 7$ $-2x > 2$ $x < -1$	**M1**

5b.

$2x - 5 > x - \frac{5}{2}$ $2x - x > 5 - \frac{5}{2}$ $x > \frac{5}{2}$	**B1**
$-(2x - 5) > x - \frac{5}{2}$ $-3x > -\frac{15}{2}$ $-x > -\frac{5}{2}$ $x < \frac{5}{2}$	**M1**
Set Notation: $\{x : x < \frac{5}{2}\} \cup \{x : x > \frac{5}{2}\}$	**M1**

6a.

$\lvert t \rvert = 3$, therefore, $t = 3$ or $t = -3$.	**B1**
When $t = 3$, $\lvert 2t - 1 \rvert = \lvert 2(3) - 1 \rvert = \lvert 5 \rvert = 5$	**M1**
When $t = -3$, $\lvert 2t - 1 \rvert = \lvert 2(-3) - 1 \rvert = \lvert -7 \rvert = 7$	**M1**

6b.

$(x - \sqrt{2})^2 > (x + 3\sqrt{2})^2$	**B1**
$x^2 - 2\sqrt{2}x + 2 > x^2 + 6\sqrt{2}x + 18$ $-8\sqrt{2}x > 16$	**M1 M1**
$x < -\sqrt{2}$	**M1**