

# A-Level Unit Test: Algebra and Functions

## Simultaneous Equations



1. Solve the simultaneous equations

$$\begin{aligned}x + y &= 2 \\ 4y^2 - x^2 &= 11\end{aligned}\quad (7)$$

2. Solve the simultaneous equations:

$$\begin{aligned}y + 4x + 1 &= 0 \\ y^2 + 5x^2 + 2x &= 0\end{aligned}\quad (7)$$

3. Find the co-ordinates of the points where the circle  $C$  with equation  $x^2 + y^2 - 2x = 19$  meets the line  $L$  with equation  $y = 3x - 1$ .

(5)

4. Find the pair of values  $(x, y)$  which satisfy the simultaneous equations

$$\begin{aligned}x^2 + 2xy + y^2 &= 9 \\ x - 3y &= 1\end{aligned}\quad (7)$$

5a. Sketch the curve  $y = 12 - x - x^2$  giving the co-ordinates of all intercepts with the axes.

(5)

b. Solve the inequality  $12 - x - x^2 > 0$

(2)

c. Find the co-ordinates of the points of intersection of the curve  $y = 12 - x - x^2$  and the line  $3x + y = 4$ .

(5)

**Total marks: 38**

# Mark Scheme

1.

$x + y = 2 \rightarrow y = 2 - x$	<b>M1</b>
$4(2 - x)^2 - x^2 = 11$ $4(2 - x)(2 - x) - x^2 = 11$ $4(4 + x^2 - 4x) - x^2 = 11$	<b>M1</b>
$16 + 4x^2 - 16x - x^2 - 11 = 0$ $3x^2 - 16x + 5 = 0$	<b>M1</b>
$(3x - 1)(x - 5) = 0$ $3x - 1 = 0$ $x - 5 = 0$	<b>M1</b>
$3x - 1 = 0, x = \frac{1}{3}$ $x - 5 = 0, x = 5$	<b>M1</b>
$x = \frac{1}{3}, y = 2 - \frac{1}{3} = \frac{5}{3}$ $x = 5, y = 2 - 5 = -3$ Solutions: $x = \frac{1}{3}, y = \frac{5}{3}$ $x = 5, y = -3$	<b>M1 M1</b>

2.

$y + 4x + 1 = 0 \rightarrow y = -4x - 1$	<b>M1</b>
$(-4x - 1)^2 + 5x^2 + 2x = 0$ $(-4x - 1)(-4x - 1) + 5x^2 + 2x = 0$	<b>M1</b>
$16x^2 + 1 + 8x + 5x^2 + 2x = 0$ $21x^2 + 10x + 1 = 0$	<b>M1</b>
$(7x + 1)(3x + 1) = 0$	<b>M1</b>
$7x + 1 = 0, x = -\frac{1}{7}$ $3x + 1 = 0, x = -\frac{1}{3}$	<b>M1</b>
$x = -\frac{1}{7}, y = -4(-\frac{1}{7}) - 1 = -\frac{3}{7}$ $x = -\frac{1}{3}, y = -4(-\frac{1}{3}) - 1 = \frac{1}{3}$ Solutions: $x = -\frac{1}{7}, y = -\frac{3}{7}$ $x = -\frac{1}{3}, y = \frac{1}{3}$	<b>M1 M1</b>

3.

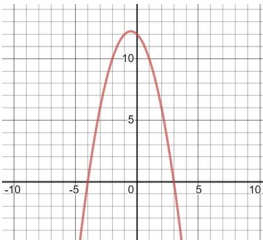
$x^2 + (3x - 1)^2 - 2x = 19$ $x^2 + (3x - 1)(3x - 1) - 2x - 19 = 0$	<b>M1</b>
$x^2 + 9x^2 - 6x + 1 - 2x - 19 = 0$ $10x^2 - 8x - 18 = 0$ $5x^2 - 4x - 9 = 0$	<b>M1</b>
$(5x - 9)(x + 1) = 0$ $5x - 9 = 0, x = \frac{9}{5}$ $x + 1 = 0, x = -1$	<b>M1</b>
$x = \frac{9}{5}, y = 3(\frac{9}{5}) - 1 = \frac{22}{5}$ $x = -1, y = 3(-1) - 1 = -4$ Solution: $x = \frac{9}{5}, y = \frac{22}{5}$ $x = -1, y = -4$	<b>M1 M1</b>



4.

$x - 3y = 1 \rightarrow x = 1 + 3y$ $(1 + 3y)^2 + 2y(1 + 3y) + y^2 - 9 = 0$	<b>M1</b>
$(1 + 3y)(1 + 3y) + 2y + 6y^2 + y^2 - 9 = 0$ $1 + 9y^2 + 6y + 2y + 6y^2 + y^2 - 9 = 0$	<b>M1</b>
$16y^2 + 8y - 8 = 0$ $2y^2 + y - 1 = 0$	<b>M1</b>
$(2y - 1)(y + 1) = 0$	<b>M1</b>
$2y - 1 = 0, y = \frac{1}{2}$ $y + 1 = 0, y = -1$	<b>M1</b>
$y = \frac{1}{2}, x = 1 + 3(\frac{1}{2}) = \frac{5}{2}$ $y = -1, x = 1 + 3(-1) = -2$ Solutions: $(\frac{5}{2}, \frac{1}{2})$ and $(-2, -1)$	<b>M1 M1</b>

5a.

$y = 12 - x - x^2$ when $x = 0, y = 12$ when $y = 0, 12 - x - x^2 = 0 \rightarrow x^2 + x - 12 = 0$	<b>M1</b>
$(x + 4)(x - 3) = 0$ $x = -4$ $x = 3$	<b>M1 M1</b>
<div style="display: flex; align-items: center;">  <div style="margin-left: 20px;"> <b>M1</b> Shape  <b>M1</b> Crossing at <math>x = -4</math> and <math>3</math>  <math>y = 12</math> </div> </div>	

5b.

From the graph, $12 - x - x^2 > 0$ $-4 < x < 3$	<b>M1</b> <b>M1</b>
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5c.

$3x + y = 4 \rightarrow y = 4 - 3x$ $y = 12 - x - x^2 = 4 - 3x$ $12 - x - x^2 - 4 + 3x = 0$	<b>M1</b>
$x^2 + x - 3x - 12 + 4 = 0$ $x^2 - 2x - 8 = 0$	<b>M1</b>
$(x - 4)(x + 2) = 0$ $x = 4$ $x = -2$	<b>M1</b>
$x = 4, y = 4 - 3(4) = -8$ $x = -2, y = 4 - 3(-2) = 10$ Solutions: $(4, -8)$ and $(-2, 10)$	<b>M1 M1</b>