A-Level Unit Test: Algebra and Functions

Index Laws & Surds

1. a) Write down the value of $64^{\frac{1}{6}}$ **(1)**

b) Simplify fully $(64x^6)^{-\frac{2}{6}}$ **(3)**

2. Express 16^{2x+4} in the form 4^y, stating y in terms of x **(2)**

3. Given that $y = 3^x$,

a) Express 9^x in terms of y **(1)**

b) Hence or otherwise, solve $(9^x) - 3^x + 1 = 0$ **(4)**

4. Solve, $64^{4x-5} = 16^{6x-10}$ **(3)**

5. a) Simplify $\sqrt{50} + \sqrt{32} - \sqrt{128}$, giving your answer in the form $a\sqrt{2}$ where a is an integer. **(2)**

b) Hence or otherwise, simplify, $\frac{12\sqrt{6}}{\sqrt{50}+\sqrt{32}-\sqrt{128}}$, giving your answer in the form $b\sqrt{c}$, where b and c are integers and $b \neq 1$

(3)

6. a) Write $\sqrt{80}$ in the form $c\sqrt{5}$, where c is a positive constant.

A rectangle R has a length of $(1 + \sqrt{5})$ cm and an area of $\sqrt{80}$ cm^2 **(1)**

b) Calculate the width of R in cm. Express your answer in the form $p + q\sqrt{5}$, where p and q are integers to be found.

(3)

7. Show that $\frac{2}{\sqrt{12}-\sqrt{8}}$ can be written in the form $\sqrt{a} + \sqrt{b}$, where a and b are integers **(5)**

Total marks: 28

Mark Scheme

1a.

$64^{\frac{1}{6}} = \sqrt[6]{64} = 2$	**M1**

1b.

$(64x^6)^{-\frac{2}{6}} = (64)^{-\frac{2}{6}}(x^6)^{-\frac{2}{6}}$	**M1**
$= \frac{1}{4}x^{-2}$	**M1** **M1**

2.

$16^{2x+4} = 4^{2(2x+4)}$	**M1**
$= 4^{4x+4}$ Therefore, $y = 4x + 4$	**M1**

3a.

$9^x = 3^{2x} = 3^x \times 3^x$ $= y \times y$ $= y^2$	**M1**

3b.

$9y^2 - 240y - 81 = 0$ $(3y - 81)(3y + 1) = 0$	**M1**
$3y - 81 = 0$, $y = 27$ $3y + 1 = 0$, $y = -\frac{1}{3}$	**M1**
$3^x = 27$, $x = 3$	**M1**
$3^x = -\frac{1}{3}$, no solution	**M1**

4.

$64^{4x+2} = 16^{6x-10}$ $4^{4(4x+2)} = 4^{2(6x-10)}$	**M1**
$4^{16x+8} = 4^{12x-20}$	**M1**
$16x + 8 = 12x - 20$ $4x = -28$ $x = -7$	**M1**

5.

$\sqrt{50} + \sqrt{32} - \sqrt{128}$ $= \sqrt{25}\sqrt{2} + \sqrt{16}\sqrt{2} - \sqrt{64}\sqrt{2}$ $= 5\sqrt{2} + 4\sqrt{2} - 8\sqrt{2}$	**M1**
$= \sqrt{2}$	**M1**

5b.

$\frac{12\sqrt{6}}{\sqrt{50}+\sqrt{32}-\sqrt{128}} = \frac{12\sqrt{6}}{\sqrt{2}}$	**M1**
$\frac{12\sqrt{6}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{12\sqrt{12}}{2}$	**M1**
$= 6\sqrt{12}$ $= 6\sqrt{4}\sqrt{3}$ $= 12\sqrt{3}$	**M1**

6a.

$\sqrt{80} = \sqrt{16} \times \sqrt{5} = 4\sqrt{5}$ $c = 5$	**M1**

6b.

Width $= \frac{\sqrt{80}}{1+\sqrt{5}} \times \frac{1-\sqrt{5}}{1-\sqrt{5}}$	**M1**
$= \frac{\sqrt{80}-\sqrt{400}}{1-5} =$	**M1**
$= \frac{4\sqrt{5}-20}{-4} = 5 - \sqrt{5}$ $p = 5$, $q = -1$	**M1**

7.

$\left\{\frac{2}{\sqrt{12}-\sqrt{8}}\right\} = \frac{2}{\left(\sqrt{12}-\sqrt{8}\right)} \times \frac{\left(\sqrt{12}+\sqrt{8}\right)}{\left(\sqrt{12}+\sqrt{8}\right)}$	**M1**
$= \frac{\left\{2\left(\sqrt{12}+\sqrt{8}\right)\right\}}{12-8}$	**M1**
$= \frac{2\left(2\sqrt{3}+2\sqrt{2}\right)}{12-8}$	**M1 M1**
$= \sqrt{3}+\sqrt{2}$	**M1**