A-Level Unit Test: Algebra and Functions

Graph Functions and Transformations

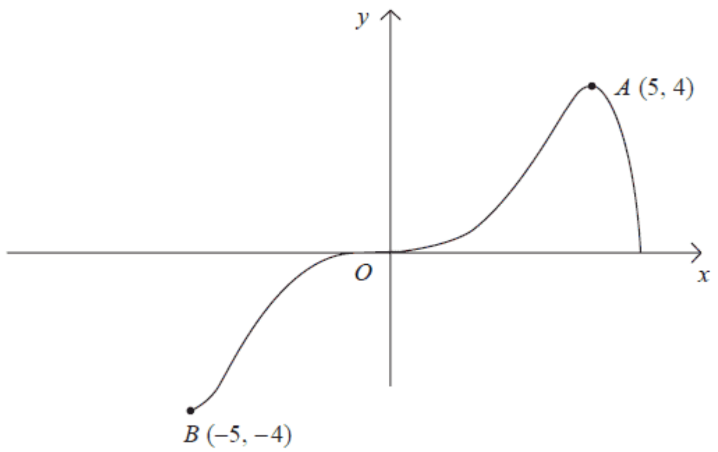
1. Figure 1 shows a sketch of the curve with equation $y = f(x)$. The curve passes through the origin O and the points (5, 4) and B (-5, -4). In separate diagrams, sketch the graph with equation.

a) $y = f(x) + 3$ **(3)**

b) $y = f(x - 9)$ **(3)**

On each sketch, show the co-ordinates of the points corresponding to A and B.

c) The graph has a transformation of $y = 2f(x + 1)$. State the new coordinates of A and B. **(2)**



2. $f(x) = (x + 3)(x - 1)^2$

a) Sketch the curve $y = f(x)$, showing the points of intersection with the coordinate axis. **(3)**

b) Find the equation of $y = f(x + 2)$ in the form $y = (x + a)(x + b)^2$ **(2)**

3. Sketch the graph of $y = \frac{1}{x} + 2$, showing the points of intersection with the co-ordinate axis and stating the equations of any asymptotes. **(3)**

4. $f(x) = x^3 + 4x^2 - 5x$

a) Sketch the curve $y = f(x)$, showing the points of intersection with the co-ordinate axis.

b) Showing the coordinates of the points of intersection with the coordinate axis, sketch on separate diagrams the curves:

i. $y = f(2x)$ **(2)**

ii. $y = 3f(x)$ **(2)**

5a. Sketch on the same diagram the curve of $y = x^2 + 5x$ and $y = -\frac{1}{x}$ **(4)**

b. State, giving a reason, the number of real solutions to the equation $x^2 + 5x + \frac{1}{x} = 0$ **(2)**

6. Figure 2 shows a sketch of the curve with equation $y = f(x)$ where,

$$f(x) = \frac{x}{x-2},\ x \neq 2$$

The curve passes through the origin and has two asymptotes, with equations $y = 1$ and $x = 2$, as shown in Figure 1.

a) Sketch the curve with equation $y = f(x - 1)$ and state the equations of the asymptotes of this curve. **(4)**

b) Find the coordinates of the points where the curve with equation $y = f(x - 1)$ crosses the coordinate axes. **(4)**

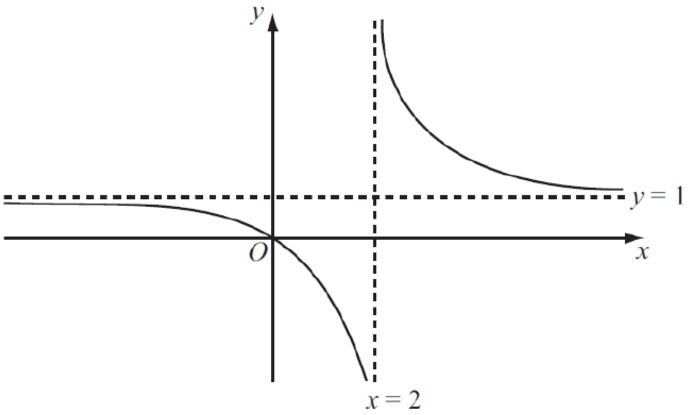


Figure 2

Total marks: 34

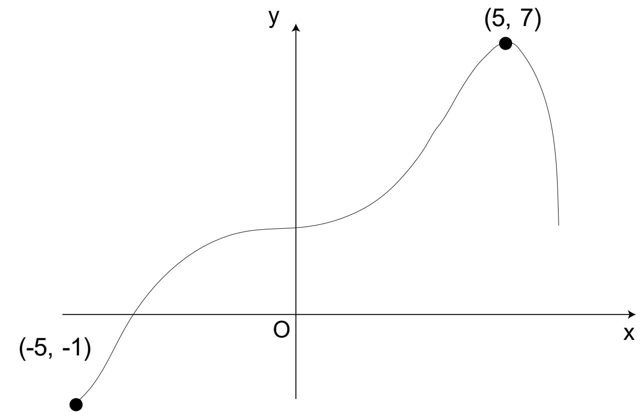
Jethwa Maths

Mark Scheme

1a.

$y = f(x) + 3 \rightarrow$ graphs moves up by 3 units

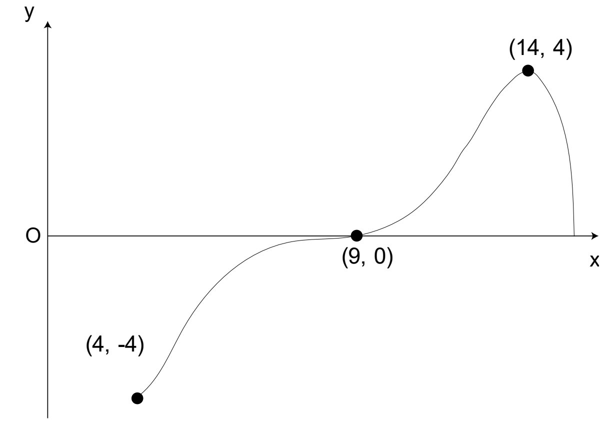
Shape **M1**
A: (5, 7) **M1**
B: (-5, -1) **M1**



1b.

$y = f(x - 9) \rightarrow$ graphs 9 units to the right

Shape **M1**
A: (14, 4) **M1**
B: (4, -4) **M1**

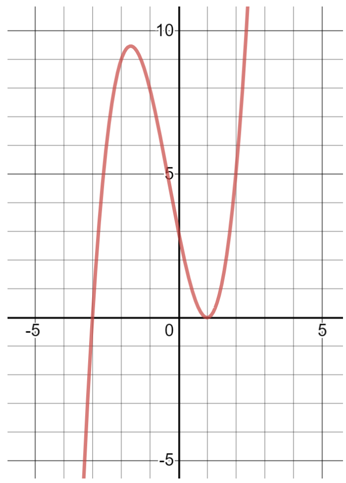


1c.

$y = 2f(x + 1) \rightarrow x$ coordinates 1 unit left, y coordinates doubled. A: (4, 8) B: (-6, -8)	**M1 M1**

2a.

$f(x) = (x + 3)(x - 1)^2$
Points of intersection on x axis: (-3, 0) and (1, 0) **M1**
Point of intersection on y axis: (0, 3) **M1**
Shape: **M1**



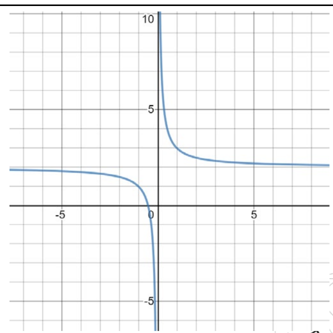
2b.

$y = f(x + 2) \rightarrow$ graph moves 2 units to the left New equation: $f(x) = (x + 5)(x + 1)^2$	**M1 M1**

3.

$y = \frac{1}{x} + 2 \rightarrow y = \frac{1}{x}$ graph shifted upwards by two units.

Asymptotes: $y = 2$ **M1**
Axis cuts: $(-\frac{1}{2}, 0)$ **M1**
Shape **M1**

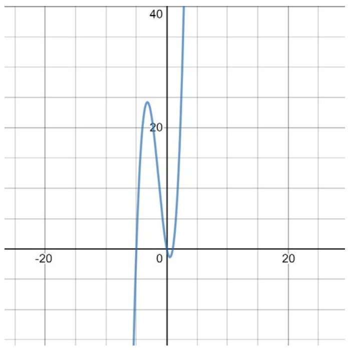


4a.

$f(x) = x^3 + 4x^2 - 5x$
$f(x) = 0$, $x^3 + 4x^2 - 5x = 0$
$x(x + 5)(x - 1)$ **M1**
Roots = 0, -5 and 1 **M1**

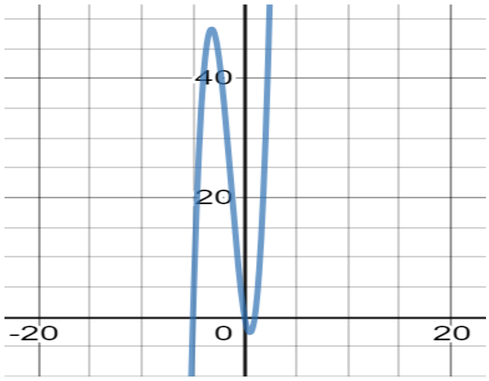
When $x = 0$, $y = 0$ **M1**

Shape **M1**



4bi.

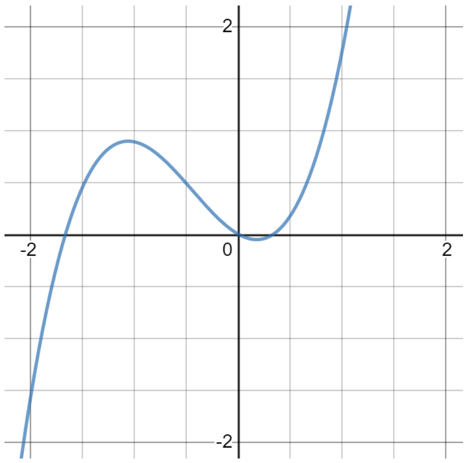
$y = f(2x)$ graph is stretched by a scale factor of 2.
Roots remain the same: 0, -5, 1



4bii.

$y = 3f(x) \longrightarrow$ graph is shrunk by a scale factor of $\frac{1}{3}$. x – coordinates are multiplied by a third.

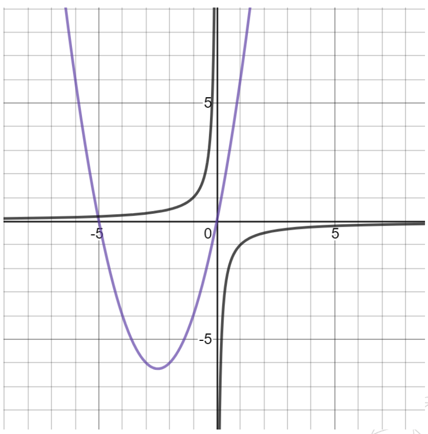
Roots: $(-\frac{5}{3}, 0)$ $(0, 0)$ $(\frac{1}{3}, 0)$



5a.

$y = x^2 + 5x = x(x + 5)$
Roots at (0, 0) and (-5, 0) **M1**
Y intercept at (0, 0) **M1**
Shape **M1**

$y = -\frac{1}{x}$
No roots **M1**
No intercept **M1**
(Asymptotes at $x = 0$, $y = 0$)
Shape **M1**

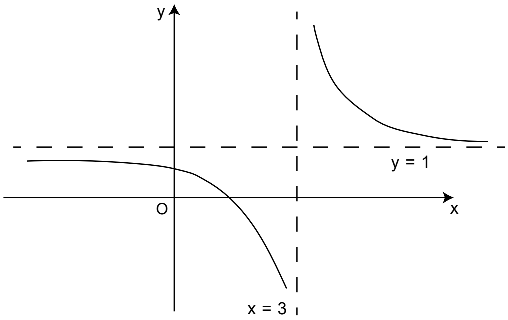


5b.

There is one real solution	**M1**
As the graphs cross once	**M1**

6a.

$y = f(x-1) \rightarrow$ graph shifts 1 unit to the right **M1**
Asymptotes: $x = 3$, $y = 1$ **M1 M1**
Shape **M1**



6b.

$f(x-1) = \frac{x-1}{x-1-2} = \frac{x-1}{x-3}$	**M1** **M1**
When $x = 0$, $y = \frac{1}{3}$ When $y = 0$, $\frac{x-1}{x-3} = 0$, $x = 1$ Co-ordinates cutting the axes: $(0, \frac{1}{3})$ and $(1, 0)$	**M1 M1**