A-Level Unit Test: Algebra and Functions

Index Laws & Surds

1. Find the set of value of x for which

a) $3x - 7 < 3 - x$ **(2)**

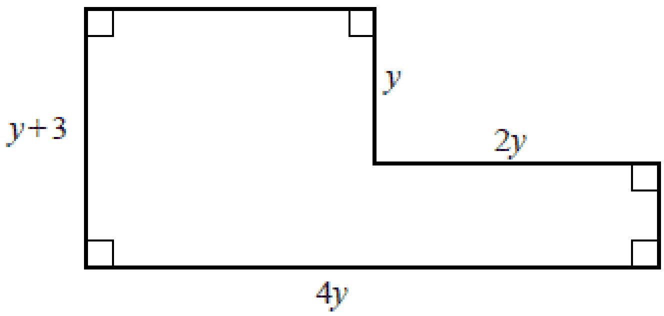
b) $x^2 - 9x \leq 36$ **(4)**

c) Both, $3x - 7 < 3 - x$ and $x^2 - 9x \leq 36$ **(1)**

2a) A rectangular tile has length $4x$ cm and width $(x + 3)$ cm. The area of the rectangle is less than 112 cm^2. By writing down and solving an inequality, determine the set of possible values of x.

(6)

b) A second rectangular tile of length $4y$ cm and width $(y + 3)$ has a rectangle of length $2y$ cm and width y cm removed from one corner as shown in the diagram.



Given that the perimeter of this tile is between 20 cm and 54 cm, determine the set of possible values of y. **(5)**

3. The equation $(k + 3)x^2 + 6x + k = 5$, where k is a constant, has two distinct real solutions for x.

a) Show that k satisfies, $k^2 - 2k - 24 < 0$ **(4)**

b) Hence find the set of possible values of k **(3)**

4. The equation $x^2 + kx + 8 = \text{k}$ has no real solutions for x. Find the set of possible values for k. **(6)**

5a. Express $2x^2 - 20x + 49$ in the form $p(x - q)^2 + r$ **(4)**

5b. State the co-ordinates of the vertex of the curve $y = 2x^2 - 20x + 49$ **(2)**

6. Given than $3x^2 + px + 16 = q(x - 2)^2 + r$ for all values of x, find the values of the constants p, q and r. **(4)**

7. $\text{f}(x) = x^2 + (k + 3)x + k$ where k is a real constant.

a) Find the discriminant of $\text{f}(x)$ in terms of k **(2)**

b) Show that the discriminant of $\text{f}(x)$ can be expressed in the form $(k + a)^2 + b$, where a and b are integers to be found.

(2)

c) Show that, for all values of k, the equation $f(x) = 0$ has real roots. **(2)**

8. The straight line with equation $y = 3x - 7$ does not cross or touch the curve with equation $y = 2px^2 - 6px + 4p$, where p is a constant.

a) Show that $4p^2 - 20p + 9 < 0$. **(4)**

b) Hence, find the set of possible values of p. **(4)**

Total marks: 53

Jethwa Maths

Mark Scheme

1a.

$3x - 7 < 3 - x$	**M1**
$4x < 10$	**M1**
$x < \frac{5}{2}$	**M1**

1b.

$x^2 - 9x \leq 36$	**M1**
$x^2 - 9x - 36 \leq 0$	**M1**
$(x - 12)(x - 3)$ $x = 12, x = 3$	**M1**
$3 \leq x \leq 12$	**M1**

1c.

$3 \leq x < \frac{5}{2}$	**M1**

2a.

$4x(x + 3) < 112$	**M1 M1**
$4x^2 + 12x - 112 < 0$ $x^2 + 3x + 28 < 0$	**M1**
$(x + 7)(x - 4) < 0$	**M1**
$x = -7, x = -4$	**M1**
$-7 < x < -4$	**M1**

2b.

Perimeter $= y + 3 + 2y + y + 2y + 3 + 4y = 10y + 6$	**M1**
$20 < 10y + 6 < 54$	**M1**
$20 < 10y + 6$ $y < \frac{7}{5}$	**M1**
$10y + 6 < 54$ $y < \frac{24}{5}$	**M1**
$\frac{7}{5} < y < \frac{24}{5}$	**M1**

3a.

$(k + 3)x^2 + 6x + k = 5$ $(k + 3)x^2 + 6x + k - 5 = 0$ $(k + 3)x^2 + 6x + (k - 5) = 0$	**M1**
Equation has two distinct real solutions, $b^2 - 4ac > 0$ $a = k + 3, b = 6, c = k - 5$	**M1**
$6^2 - (4)(k+3)(k - 5) > 0$ $36 - 4(k^2 - 2k - 15) > 0$ $36 - 4k^2 + 8k + 60 > 0$	**M1**
$-4k^2 + 8k + 96 > 0$ $-k^2 + 2k + 24 > 0$ $k^2 - 2k - 24 < 0$	**M1**

3b.

$k^2 - 2k - 24 < 0$	**M1**
$(k - 6)(k + 4) = 0$ $k = 6, k = -4$	**M1**
$-4 < k < 6$	**M1**

4.

$x^2 + kx + 8 - k = 0$ $x^2 + kx + (8 - k) = 0$	**M1**
Equation has no real real solutions, $b^2 - 4ac < 0$ $a = 1, b = k, c = 8 - k$	**M1**
$k^2 - (4)(1)(8 - k) < 0$ $k^2 - 4k - 32 < 0$	**M1** **M1**
$(k - 8)(k + 4) = 0$ $k = 8, k = -4$	**M1**
$-4 < k < 8$	**M1**

5a.

$2x^2 - 20x + 49 = 2(x^2 - 10x) + 49$	**M1**
$= 2[(x - 5)^2 - 25] + 49$	**M1**
$= 2(x - 5)^2 - 50 + 49$	**M1**
$= 2(x - 5)^2 - 1$	**M1**

5b.

From part a) $2(x - 5)^2 - 1$ Vertex: (5, -1)	**M1 M1**

6.

$3x^2 + px + 16 = q(x - 2)^2 + r$ $= q(x - 2)(x - 2) + r$ $= q(x^2 - 4x + 4) + r$	**M1**
$= qx^2 - 4qx + (4q + r)$	**M1**
Comparing coefficients: x^2: $q = 3$	**M1**
x: $p = -4q$, $p = -12$ constants: $q + r = 16$, $r = 13$	**M1**

7a.

$a = 1, b = k + 3, c = k$ $b^2 - 4ac = (k + 3)^2 - (4)(1)(k)$	**M1**
$= k^2 + 6k + 9 - 4k$ $= k^2 + 2k + 9$	**M1**

7b.

$k^2 + 2k + 9 = (k + 1)^2 - 1 + 9$	**M1**
$= (k + 1)^2 + 8$ $a = 1, b = 8$	**M1**

7c.

$(k + 1)^2$ will always give a positive value	**M1**
$(k + 1)^2 + 8$ positive term plus a positive term will always be ≥ 0, therefore the function has two real roots.	**M1**

8a.

$2px^2 - 6px + 4p = 3x - 7$ $2px^2 - 6px + 4p - 3x + 7 = 0$ $2px^2 - (6p - 3)x + (4p + 7) = 0$	**M1**
Curve and line do not cross, therefore, $b^2 - 4ac < 0$ $a = 2p$, $b = 6p - 3$, $c = 4p + 7$ $(6p - 3)^2 - 4(2p)(4p + 7) < 0$	**M1**
$(6p - 3)(6p - 3) - 8p(4p + 7) < 0$ $36p^2 - 36p + 9 - 32p^2 - 56p < 0$	**M1**
$4p^2 - 20p + 9 < 0$	**M1**

8b.

$4p^2 - 20p + 9 < 0$	**M1**
$(2p - 9)(2p - 1) = 0$	**M1**
$2p - 9 = 0$, $p = \frac{9}{2}$ $2p - 1 = 0$, $p = \frac{1}{2}$	**M1**
$\frac{1}{2} < p < \frac{9}{2}$	**M1**